ESTIMATING BIRD/AIRCRAFT COLLISION PROBABILITIES AND RISK UTILIZING SPATIAL POISSON PROCESSES

GRADUATE RESEARCH PAPER

Brady J. Vaira, Major, USAF
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DEPARTMENT OF THE AIR FORCE
AIR UNIVERSITY

AIR FORCE INSTITUTE OF TECHNOLOGY

Wright-Patterson Air Force Base, Ohio

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Brady J. Vaira, BS, MS

Major, USAF

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Approved:

//signed//	31 May 2012
Dr. Jeffery Cochran (Chairman)	date

Abstract

Aircraft collisions with avian species are a serious safety problem as well as a serious economic issue. Aircraft / bird strikes have resulted in 33 fatalities, the loss of 39 aircraft, and damages to aircraft in excess of $820M for the United States Air Force. The objective of this paper is to create a closed form mathematical model that estimates the probability of a bird / aircraft collision and provides a risk score that can be utilized to underpin decisions made by planners and pilots.

The major components of the model are the spatial Poisson process, the extended spatial Poisson process, a gamma distribution of bird altitudes, a relative risk score, a standardized risk score scale, and a risk filtering and ranking method. The spatial Poisson process allows for an independent distribution of birds within a bounded area. The extended spatial Poisson process accounts for the removal of birds from calculations within the bounded area after they have been encountered. The gamma distribution models the distribution of specific bird altitude bands within a bounded area. The relative risk score is a weighted risk score for 19 different species of birds that an aircraft might encounter. The standardized scale aggregates all risk scores over all the bird species and then calculates the value in a 0 to 10 scale. The risk filtering and ranking model combines the effects of a hit with the likelihood of a hit and displays the result in a graphic. The overall model that combines these components and calculates the output is an original contribution to the field of aircraft / avian collision models.

Exercising the model reveals significant factors that influence the risk score associated with flying in a particular area. They are the total number of birds in the bounded region, the mix of species within the bounded region, the size of the aircraft, and the gamma height distribution of the birds within the bounded region.

Knowing the gamma height distribution for the specific birds in an operations area (AO) can provide more fidelity to the planner. In fact, in several scenarios where the same number and species of birds for an AO was used, the difference in the overall aggregated risk score was twice as high as the score that was calculated when the gamma height distribution was not known. Additionally, when there were densely populated altitude bands of birds in the operations area, avoiding these bands cut the overall risk score by up to 50%. This is very useful information for decision makers to have when they are planning the specifics of their operations.

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1. Introduction

1.1. Background

Random collisions of relatively small numbers of entities in very large, yet bounded, spaces are rare, but not impossible. In fact, consider the time period spanning 15 January 2009 to 4 February 2009. During this 20 day period, the U.S. commercial Iridium spacecraft was hit by a defunct and out of control Russian satellite at an altitude of about 800km (500 miles) over Siberia (BBC NEWS 2009); additionally, a Royal Navy nuclear submarine was involved in a collision with a French nuclear sub in the middle of the Atlantic Ocean (BBC NEWS 2009) and finally, US Airways flight 1549 made an emergency water landing as a result of a bird strike (a large flock of Canada geese disabled both engines) in the Hudson River in New York (U.S. Airways 2009).

While the first two cases of collisions are quite an anomaly, aircraft/wildlife strikes in general do happen frequently. According to the Federal Aviation Administration (FAA), which has maintained a wildlife strike database since 1990, there have been 121,000 (civil and U.S. Air Force) wildlife strikes between 1990 and 2010. The U.S. Air Force (USAF) has been collecting statistics since 1985 and has recorded 95,383 wildlife strikes spanning the period from 1985 to 2011, resulting in: 33 fatalities, the loss of 39 aircraft, and damages to aircraft in excess of $820M (U.S. Air Force Safety Center 2012).

According to the FAA, 92% of the bird strikes to commercial aircraft occur at or below 3,500 ft above ground level (AGL). Using this same threshold, the USAF has recorded 96.72%

of its bird strikes below 3,500 ft AGL. In order to narrow down scope of the problem set, statistics on USAF wildlife strikes by phase of operations were examined. The following table highlights the number of bird strikes for the different phases of operation (U.S. Air Force Safety Center 2012).

Table 1 USAF Wildlife Strikes by Phase of Operations (1995-2011)

Phase of Flight	Cost ($MM)	% of Total	Count	% of Total
Takeoff/Initial Climb	137.0	32.2%	7299	12.3%
Enroute/Air Work/Air-to-Air/Air Refueling	19.2	4.5%	2830	4.8%
Flight Demonstration	1.9	0.4%	27	0.1%
Low Level/Air-to-Ground/Air Delivery	174.2	40.9%	6949	11.7%
Hover	0.0	0.0%	10	0.0%
Traffic Pattern/Go-Around	31.4	7.4%	7618	12.8%
Initial Approach/Final Approach/Landing	46.8	11.0%	16048	26.9%
Parked/Ground Ops	3.6	0.8%	423	0.7%
Unknown	12.1	2.8%	18400	30.9%

These statistics clearly do not support the "big sky, little plane" theory and as such, there are extensive mitigation and alert/advisory models and systems in place throughout the world to help reduce the occurrence of aircraft/bird collisions. Both active and passive mitigation & abatement methods/systems are employed around airfields in order to diminish the possibility of an aircraft/bird strike. These methods include: pyrotechnics, bioacoustics, exclusion, propane gas cannons, falconry, dogs, radio controlled crafts, grass habitat management, sanitary landfill management, agricultural leases, and depredation (U.S. Air Force 2004).

However, for most low level routes, air-to-ground, and air delivery operations, mitigation and abatement methods and/or systems are not available and as Table 1 indicates, the largest dollar cost and a high total count is evident, even though a smaller percentage of flights fly this profile. To assist in reducing the probability of a bird strike in this phase of operation, advisory models and systems are in place to assist with planning and real-time avoidance. These advisory and alert systems vary on methods used (historical, real-time, algorithms, etc) and provide pilots and planners with different levels of information and fidelity.

1.2. Problem Statement

The purpose of this research is to create a closed form mathematical model to determine the probability of an aircraft/wildlife collision encounter (strike) in a defined space (e.g. on a segment of a Low Level route/Air-to-Ground/Air Delivery) with a set of given parameters and then give weighted risk associated with the encounter. The output analysis will be presented via a Decision Support System (DSS) in order to underpin decisions by pilots and planners when coupled with existing alert and advisory systems.

1.3. Research Objectives

To determine the probability of an aircraft bird encounter that has the possibility of leading to a bird strike, the following research objectives are considered:

- Study the systems that provide avian detection, alerts and advisories; discuss the level of fidelity they provide.
- Determine avian characteristics, geographical and environmental conditions, and other factors that drive avian population densities and spatial patterns.
- Conduct research on Spatial Poisson Processes and relate them to aircraft/avian encounters for a defined space.
- Highlight and discuss existing aircraft/avian collision mathematical models.
- Conduct research on Extended Spatial Poisson Processes and how they relate to the encounter model and techniques for solving matrix exponentiation.

2. Literature Review

2.1. Overview

This section discusses the completed research that support the research objectives listed above.

There is a vast amount of research that has been completed on bird strikes. In fact there are numerous international committees that meet to discuss trends, best practices, research, detection and advisory models, and other related topics. Additionally, there are many books, journals and newsletters that have been published which discuss this issue. Finally, there has been a marked increase in the technology developed for avian detection and advisory. All of this is aimed at reducing the number and severity of bird strikes throughout the world as both the number of aircraft operations and large bird species populations increases. For example, in North America, 13 of the 14 largest (>3.6 kg body mass) bird species have shown significant population increases in the past 40 years. In fact, the migratory and non-migratory population of Canadian geese (average weight of 4.2 kg) has more than quadrupled from 1.2 million to 5.5 million birds in North America from 1970 to 2008. (Dolbeer 2009).

Random collisions in $\mathbb{R}^d$- space have been studied at great length by many scholars. There are numerous areas of specific study within the three main groups of models which are: gas particle models, satellite models and historical models. This paper will look at some of the specific areas of study related to random collision. The focus will be on the Spatial Poisson Process, as it allows the most flexibility when dealing with the characteristics and behaviors of aircraft and bird interactions.

The end result of the research is to provide a Decision Support System that combines some of the ideas garnered from the review of avian characteristics, advisory & alert systems with the mathematical modeling of spatial Poisson processes. The goal is to create a model that allows for parameter inputs for 1) the area of operation (AO), 2) the type, number (and other associated characteristics) of the birds in the AO and 3) the size (wingspan) and distance of the aircraft flying through the AO and then provides a probability for the aircraft / bird encounter along with a risk value.

2.2. Avian Characteristics and Environmental Factors that Influence Populations

In order to derive avian density for a particular area, one must first discuss some fundamentals of avian movement patterns. Avian movements can be partitioned into three categories: migrating birds, commuting birds and resident birds. (FAA 2010). Within these categories, there are many factors that affect the avian population density for a specific area. The most influential factors affecting bird movements and roosting, loafing and feeding locations are wetlands locations, land use practices, water management facilities and agricultural activities (Cleary and Dolbeer 2005). Specifically, the most influential areas within each group can further be defined.

Land use practices that attract birds are:

- Waste disposal operations
- Underwater waste discharges
- Trash Transfer Stations
- Composting operations
- Fly ash disposal
- Recycling centers
- Constructions and demolition debris facilities

Water management facilities that attract birds are:

- Storm water management facilities
- Wastewater treatment facilities
- Artificial marshes
- Wastewater discharge and sludge disposal

Agricultural activities that attract birds are:

- Crop production
- Livestock production
- Aquaculture

Therefore, it is noted that the population density of birds varies considerably among regions and habitats. In general, greater numbers of bird species are attracted to areas offering more diverse food sources; an abundance of food, cover and water leads to larger numbers of birds.

The distribution and density of birds also changes with the season. In the Northern Hemisphere, bird numbers peak in the summer, after the breeding season. In the far north, Snow Geese breed in colonies of up to 150,000 pairs. Large sea-bird colonies comprised of thousands of nesting birds are found along both the eastern and western seaboards of Canada and the United States. Around the Great Lakes, on small islands, gull colonies have been documented as containing over 40,000 breeding pairs. During migration, birds of some species funnel to and congregate at key staging areas along the flyways. As a result, relatively small areas can become the temporary home to extremely high concentrations of birds, and many airports, low-level routes, operating areas and ranges are located along major migratory bird routes (Transport Canada 2004). The migratory phenomenon is worldwide. Large eastern European birds migrate to Africa annually, passing along the eastern shore of the Mediterranean Sea. Additional

European-Africa routes are over the Iberian Peninsula and the Italian Peninsula to central, southern and eastern Africa (Eschenfelder 2005).

Additionally, the time of day affects the population activity. The highest levels of daily wildlife activity normally occur +/- one hour of sunrise/sunset as birds move to and from their roosts (U.S. Air Force 2011).

Finally, the most significant influence on the altitude at which birds fly is the weather, specifically cloud cover and wind fields. Birds may fly lower when it is cloudy or, if the overcast is not too thick, they may ascend through it to reach the clear skies above. If favorable tail winds are to be found in certain altitudinal strata, birds often ascend or descend in order to take advantage of them. A comparative analysis of the influence of weather on the flight altitudes of birds was conducted by researchers. They found that flight altitudes of birds differ among species and vary greatly from day to day. Many factors, in addition to weather, may influence the flight altitudes of birds. The study utilized several different parameter inputs and regression analysis was accomplished for different species of birds in order to form a prediction model for bird altitude. Data was collected via radar to the level of individual birds and type was determined by visual or classification of wing beat frequency. Some of the input parameters were: relative humidity, min and max daily temperature, lift index, sea level pressure, wind speed, aeronautic index, boundary layer height, thermal index, vertical wind gradient and cloud cover. Different combinations of these factors explained 40% to 70% of the variance in maximum flight altitudes for the particular species studied (Shamoun-Baranes, Van Loon, et al. 2006).

2.3. Avian Detection and Advisory

Different techniques have been used to warn the pilots and flight schedulers/planners of potential bird threats.

Radar has been used extensively to monitor bird movements and warn the relevant personnel (Sodhi 2002). One of the primary radars used to monitor bird movements in the U.S. is the WSR-88D NEXRAD radar. The WSR-88R is the Doppler weather radar that is located throughout the United States, Alaska, Hawaii, and Puerto Rico. The WSR-88D can readily detect birds in the atmosphere in both clear air and precipitation mode. This weather radar provides information on movements within 124 nautical miles (NM) for a single radar station as well as regional and national scale for multiple radar sites (Gauthreaux and Belser 1998). Dokter *et al* showed that weather radar can extract near real-time bird density altitude profiles that closely correspond to the density profiles measured by dedicated bird radar (Dokter, et al. 2011). The WRS-88D is used in the Avian Hazard Advisory System which is described later in this paper.

Advisory systems provide pilots and planners a method for estimating risk associated with bird strikes. One of the vanguards of these advisory systems is the United States Bird Avoidance Model (USBAM). The USBAM is based on approximately 30 years of historic bird observation data for winter and summer distributions. These point data were/are transformed into average bird mass values and are interpolated spatially in a GIS environment for each of the bird strike relevant species with a resolution of 1 km^2. Between the winter and the summer distribution a temporal interpolation is conducted, based on diurnal and annual activity pattern, breeding success and mortality rate. The overall average mass of birds per km^2 is transformed into bird strike risk levels of low, moderate and severe. Model output is displayed in an internet map application that combines the bird strike risk level information with additional important map information for aviators.

Modeling in this case stands for a widely automated process to transform the historic bird count information into average time and space dependent bird strike risk levels. Updates according to new data are provided approx. every 2-5 years (Ruhe 2005).

The risk levels that USABAM uses describe three predicted risk classes - Low, Moderate, and Severe, which are based upon the bird mass in ounces per square kilometer. The "Moderate Zone" indicates a risk ratio that is 57-708 times the risk of the "Low Zone", while the "Severe Zone" indicates a risk ratio that is 2,503-38,647 times the risk of the "Low Zone" (United States Air Force BASH 2012).

The species data was acquired from several key datasets, including the Audubon Societies' Christmas Bird Count, the US Biologic Survey's Breeding Bird Survey, bird refuge arrival and departure data for the contiguous US, and many additional data that are specific to a particular bird species.

The United States Avian Hazard Advisory System (AHAS) is an online, near real-time, geographic information system (GIS) used for bird strike risk flight planning across the continental United States. Using NEXRAD (WSR-88D) weather radars and models developed to predict bird movement, AHAS monitors bird activity and forecasts bird strike risk as well. AHAS takes current weather data from the National Weather Service and calculates the risk large bird species present, based upon the relationships found between behavior, weather and strike rate with each species. Standard meteorological calculations are used to determine thermal depth and strength. Weather data is also used to determine when birds will initiate migration (Kelly, et al. 1999).

AHAS consists of the following elements:

- A forecast of bird migratory activity for the next twenty-four hours.
- A forecast of soaring bird activity for the next twenty-four hours.
- Near real time monitoring of bird activity with NEXRAD radar.
- Radar Data archiving for system development.

2.4. Avian Altitude Density Distributions

For bird strikes that were reported at ≤ 500 feet, passerines, gulls and terns, pigeons and doves, waterfowl, and birds of prey were the species groups most frequently struck. For strikes above 500 feet, waterfowl, gulls and terns, passerines, birds of prey, and vultures were the species groups most frequently struck. Waterfowl comprised 53% of the identified birds struck above 3,500 feet (Dolbeer, Height Distribution of Birds Recorded by Collisions with Civil Aircraft 2006). Additionally, 74% of all reported strikes were ≤ 500 feet AGL.

For the 24% of strikes above 500 feet, and using height interval as the single independent variable and number of reported strikes per interval as the dependent variable, Dolbeer determined the regression equation that gave the best correlation for reported bird strikes to elevation for bird strike data from 1990-2004. His analysis resulted in a negative exponential model with height as the independent variable. The height variable explained 99% (R^2 = 0.9891) of the variation in number of bird strikes per 1,000-foot interval (starting at 500 feet).

$$E[Strikes] = 4469.2\exp(-0.3846 * FlightAltitude) \tag{1}$$

Plotting this function against altitude gives the following:

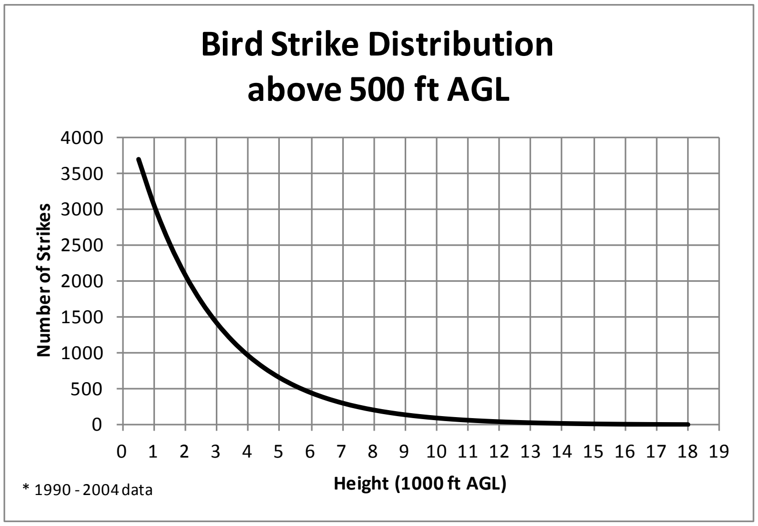


Figure 1 Bird Strike Distribution

Thus, the majority of strikes happen below 3500 feet AGL. However, this data does not explicitly give the distribution of avian altitudes.

To better understand avian altitude stratification, specific research for an area is required. Bird populations are constantly changing in response to various anthropogenic as well as natural factors. It is estimated that the longevity of the predictions by bird avoidance models (BAMs) to be in the order of 5–10 yr. Therefore, a model update is recommended approximately once every 5 years (Shamoun-Baranes, et al. 2008).

In the Netherlands, a multidisciplinary team developing a Bird Avoidance Model for Northwest Europe captured data on 60 different species of birds in order to model bird distribution and flight altitude predictors and distributions (Shamoun-Baranes, Sierdsema, et al. 2005). Data was collected via Hollandse Signaal Apparaten (HSA; Hengelo, the Netherlands) midlife update (MLU)-Flycatcher tracking radar. Identification and further refinement was collected using two methods: 1) visual identification using a video camera with a 300-mm lens mounted parallel to the tracking radar, and 2) the classification of wing beat frequencies (Shamoun-Baranes, Van Loon, et al. 2006).

Multiple linear regression models were built by fitting explanatory variables to maximum hourly flight altitude as the response variable. Explanatory variables included several meteorological variables such as temperature, wind speed and direction, boundary layer height, relative humidity, lifted index (a measure of atmospheric instability), time of day and time of year. Each species was analyzed separately. In order to keep models relatively simple, their models were limited to no more than four significant ($p \leq 0.05$) explanatory variables.

The following represents the estimated maximum altitude of a buzzard based upon the explanatory variables of relative humidity (RH), boundary layer height (BLH), maximum daily temperature (Tmax), and lift index (LFTX).

$$\ln(height) = \{-0.02[RH](\%) + 0.0007[BLH] + 0.077[T\max] - 0.041\{LFTX] + 5.33\} \qquad (2)$$

Additionally, the study captured the altitude distribution of the species studied. Figure 2 shows the distribution for the buzzard with an associated gamma distribution.

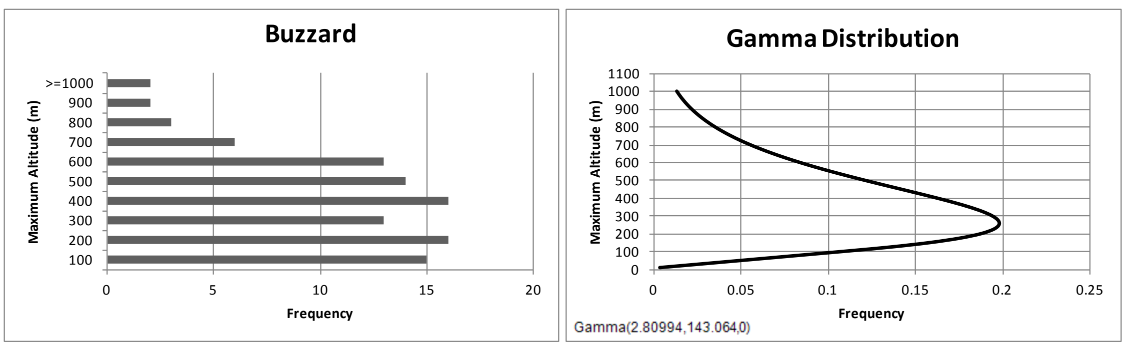


Figure 2 Buzzard Altitude Distribution

Multiple studies have estimated the best-fit parameters for the observed flight height distribution. The empirical data was fitted using maximum-likelihood parameter estimations. Various literatures (Intachat and Holloway 2000), (Stumpf, et al. 2011) and (Shamoun-Baranes, Van Loon, et al. 2006) reported that the gamma distribution (among others) was an appropriate distribution. In probability theory and statistics, the gamma distribution is a two-parameter family of continuous probability distributions, with a shape parameter "α" and a scale parameter "β" called the rate parameter.'

The model in this paper uses an independent gamma distribution (if known) for each species (if known). However, if the species of bird (but not the percentages of each species) in an AO is known and a gamma distribution of the species is known, then one must explore the ability to add gamma distribution in order to have a gamma distribution across the entire AO. If the beta parameters of the gamma distributions are equal, then the gamma distributions can be added.

If X_1, X_2,..., X_n are independent random variates with a gamma distribution having parameters (α_1,β), (α_2,β),...,(α_n,β) then $\sum_{i=1}^{n} X_i$ is distributed as gamma with parameters:

$$\alpha = \sum_{i=1}^{n} \alpha_i, \beta = \beta \tag{3}$$

If however, the beta parameters are different, then the process is not as simple as outlined above. In this situation, it is feasible to 1) use an equal percentage for the known birds in the AO and then apply the specific gamma distribution for that species of bird or 2) use an approximation of the parameters. The approximation is outlined in Thom's work fitting the gamma distribution for precipitation data. Since the *shape* (α) parameters add in convolution of gamma distributions, and the mean $\bar{x}$ is also a sufficient estimator of μ (the 1st moment) it is straightforward to determine the *scale* (β) from the summed means (Thom 1968). This gives the following estimates "g" for shape and "b" for scale:

$$g = \sum_{i=1}^{m} g_i, \tag{4}$$

$$b = \sum_{i=1}^{m} \frac{\overline{X}_i}{g} \qquad \text{where } m = \text{\# of gamma distributions}$$

2.5. Independence - Bird Flocks / Small Bird Estimates

Because radar is primarily utilized to detect and identify bird species in an area of operations common to low level routes or bombing ranges, the flocking behavior of birds can make it difficult to determine the number of birds in the flock and thus the type of bird present. One technique that many researchers utilize is the "small bird estimate" that was first utilized by Kelly (1995). Birds of each size are scaled to standardize the birds by mass into categories. The use of small bird estimates (SBE) helps to counter the problem of unknown bird numbers per radar target. A medium target on the radar screen may be a single intermediate-sized bird or a small flock of small birds. Either way it is represented in the researcher's model as the same number of SBE's. Their assumption is that it is equally hazardous to strike one intermediate-sized bird or a small flock of small-sized birds. A larger flock of small birds, a small flock of intermediate-sized birds, and an individual large bird would all be categorized as large bird targets and would be recorded as the same number of SBEs. Thus, the numbers birds per radar target, hence risk, though not completely quantifiable, is incorporated in the algorithm (Zakrajsek and Bissonette 2002).

Budgey's research indicated that there is an apparent relationship between the wingspan of a flocking bird, its nearest neighbor distance and the number of birds likely to be struck by an aircraft encountering a flock of that species (Budgey 1998). His research was limited in that not all species were studied and the number of flocks per species studied was small. If his research holds for all species, then this method of determining the number of birds in a flock and the probability of the number of bird strikes could be used to enhance current collision models. One would need to determine the probability of encountering the flock, and then determine the probability of a strike from within the flock.

For the models used in this paper, it will be assumed that the bird species identification is known either through visual, historical or SBE approximation. Flocking behavior and numbers per flock will not be utilized. If there is enough fidelity in determining birds in an AO, then each bird will be treated independently and assumed to follow a Poisson Spatial distribution.

Risk Level Ranking

All wildlife species are not equally hazardous to aviation. The hazard of a species to an aircraft is strongly related to the species mean body mass. However, it is not the only factor when modeling the hazard associated with a specific bird.

The FAA historical model looked at the hazard level for 108 bird species with 25 or more strikes, based on the percentage of strikes that have caused an adverse effect (damage and/or negative effect on flight), ranged from 0 percent for 8 species to 80 percent for snow geese (FAA 2012).

The Royal Netherlands Air Force models their bird hazard index to be equal to the relative strike sensitivity times the damage probability.

(Morgenroth 2003) modeled bird strike hazards based on behavioral specific aspects of birds. Zakrajsek and Bissonette’s (2002) model was based on military bird strike statistics and specifically focused on the number of damaging strikes and the cost associated with that damage as a criteria.

DeVault used percentage of strikes with damage, percentage of strikes with substantial damage and percentage of strikes with effect on flight (EOF) as factors in which species were ranked and a relative hazard score was calculated. For birds, they assessed effects of body mass, body density, and group size on relative hazard scores for 77 species. Additionally, they only used data from strikes that occurred at less than 500 feet AGL (DeVault, et al. 2011). Table 2 highlights the top 15 hazardous birds based upon their research and model.

Table 2 DeVault Top 15 Relative Hazard Score

Species	Total strikes reported	Composite rank	Relative hazard score	Body mass (g)	% of strikes with mult. birds
Other geese*	20	1	100	2290	60
Canada goose	776	2	76	3564	47.9
Other ducks*	77	2	78	916	46.8
Turkey vulture	159	2	73	1467	9
Double-crested	24	5	71	1674	16.7
Great horned owl	29	5	72	1309	3.4
Brown pelican	31	7	66	3348	9.7
Sandhill crane	66	8	61	5571	44.6
Glaucous-winged gull	27	9	64	1010	25.9
Wild turkey	38	9	65	5811	23.7
Bald eagle	74	11	59	4740	12.2
Great black-backed gull	20	12	53	1659	15
Osprey	77	13	53	1485	2.6
Great blue heron	132	14	51	2390	2.3
Ring-necked pheasant	45	15	47	1135	8.9

Dolbeer, Wright and Cleary created a model to rank 19 different bird species based upon 1) Damage, 2) Major Damage, and 3) Effect on Flight. Using rankings within each category, they developed a composite ranking and a relative hazard score. The relative rank of each species group was compared with every other group for the 3 categories, placing the species group with the greatest hazard rank for ≥ 2 of the 3 categories above the next greatest-ranked group. Their relative hazard score was related strongly to mean body mass for the 19 bird species groups (R^2 = 0.71, p<.01). Table 3 contains their final results (Dolbeer, Wright and Cleary, Ranking the Hazard Level of Wilflife Species to Aviation 2000).

Table 3 Risk Level Ranking

	Ranking by Criteria				
Species Group	Damage	Major Damage	Effect on flight	Composite Ranking	Relative Hazard Score
Deer	1	1	1	1	100
Vultures	2	2	2	2	63
Geese	3	3	4	3	52
Cranes	4	4	7	4	48
Osprey	6	5	3	5	50
Pelicans	5	7	5	6	44
Ducks	7	6	8	7	37
Hawks	8	15	9	9	25
Eagles	8	15	9	9	31
Rock Dove	11	8	11	10	24
Gulls	10	11	13	11	22
Herons	12	14	12	12	22
Mourning Dove	14	9	17	13	17
Owls	13	12	19	14	16
Coyote	15	17	6	15	20
American Kestrel	16	10	16	16	14
Shorebirds	17	19	14	17	12
Crows - Ravens	18	16	15	18	12
Blackbirds - Starling	19	18	18	19	9
Sparrows	20	21	20	20	4
Swallows	21	20	21	21	2

2.6. Spatial Poisson Process

Spatial point pattern data occurs frequently in a wide variety of scientific disciplines, including seismology, ecology, forestry, geography, spatial epidemiology, and material science (Møller and Waagepeterson 2006). One may think of a spatial point process as a random countable subset of a space X. Assume that $X \subseteq \mathbb{R}^d$. Typically, X will be a d-dimensional box or all of $\mathbb{R}^d$. However, it could also be X^{d-1}, the (d-1) – dimensional unit sphere (Johnson 2010). In many instances, one may only observe points in a bounded subset (window) $A \subseteq X$. For example, there may be interest in the spatial distribution of a certain species of tree in a wilderness area. Without having to count or record every tree, efforts should be concentrated on several square windows where $A_i = [a_{i0}, a_{i1}]^2$, $a_{i1} > a_{i0,}$ $A_i \cap A_j = 0,\ j \neq i$ and $A = \cup_i A_i \subset X$. One of the simplest and fundamental spatial point processes is the completely random, or spatial Poisson point process (Guttorp, Brillinger and Schoenberg 2002). Spatial Poisson processes play a fundamental role in the theory of point processes. They possess the property of "no interaction" between points or "complete spatial randomness". The points are stochastically independent and the probability of the number of points N(A) in a region A, is given by the Poisson distribution as outlined below:

Let $\Omega^X = \{\omega - (x_i)_{i=1}^N \subset X, N \in N \cup \{\infty\}\}$ denote the space of configurations on $X = \square^d, d \geq 1$.

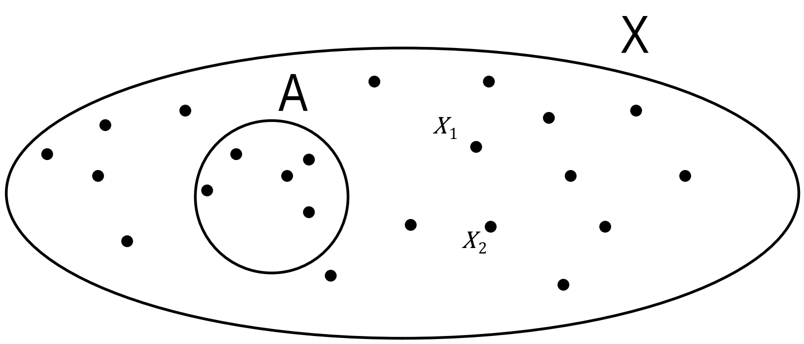


Figure 3 Spatial Poisson Process in 2-Space

Let $\omega(A) = \#\{x \in \omega : x \in A\} = \sum_{x \in \omega} 1_A(x)$.

The Poisson Probability measure $\mathbb{P}_\sigma$ with intensity *ρ(x)dx* on X satisfies

$$\mathbb{P}_\sigma\ (\omega \in \Omega^X : \omega(A) = n) = e^{-\sigma|A|} \frac{(\sigma|A|)^n}{n!}, \quad n \in \mathbb{N}$$

with $\sigma|A| = \int_A \rho(x) dA = \int_{\mathbb{R}^d} 1_A(x) \rho(x) dx$.

Usually the intensity function *ρ(x)* will be constant (homogeneous), i.e. $\rho(x) = \lambda \geq 0$, $x \in X$, where $\lambda > 0$ is called the intensity parameter:

$$\sigma|A| = \lambda * \int_A dA$$

This paper assumes that the intensity function is constant, which gives the Spatial Poisson Process distribution for the probability of "n" entities in the bounded region A (with area/volume/etc of |A|) and intensity function λ as:

$$p_n = P(\omega \in \Omega^X : \omega(A) = n) = \frac{(\lambda|A|)^n}{n!} e^{-\lambda|A|} \tag{5}$$

A practical application of the spatial Poisson process (SPP) distribution is observed in a minefield crossing (Kim 2002). In this example, the premise is that mines are buried in a field of known dimension. The location of the mines is not known, but the number of mines in this bounded area is known. The author asserts that the placement of the mines follows a SPP and they will detonate if you are with a certain distance "r = 1.5 feet" from the mine. Suppose the field is 100 ft x 200 ft and you are crossing it at its shortest distance (100 ft). The intensity of the mines is λ = 15/20000 = .00075 per square foot.

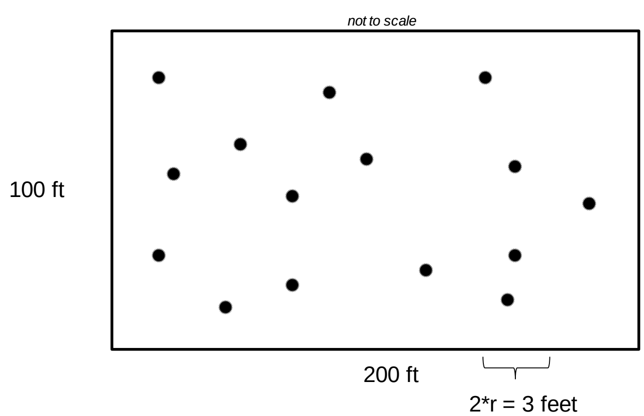


Figure 4 Minefield Example

The region |A| in this case is 3 ft x 100 ft = 300 square ft. The probability of safely transiting the minefield (probability of no strikes) is:

$$p_0 = P(\omega \in \Omega^X : \omega(A) = 0) = \frac{(0.00075 * 300)^0}{0!} e^{-0.00075*300} = 0.798516 \tag{6}$$

Thus, in this situation, there is a ~80% chance of traversing the minefield safely.

This concept can easily be extended to 3-space. In the following example, the probability of an aircraft / bird encounter (defined as a bird being within a certain proximity to the aircraft) is desired. Table 4 contains bird sightings from Dover AFB (KDOV) during the morning of 2 October, 2011.

Table 4 Dover AFB Bird Sightings

Date	Time	Zone	Species ID	Number Observed	Habitat	Behavior	Observation Method
10/2/2011	08:29:31	Airfield	Canada Geese	16	Sky	Overflight	Visual
10/2/2011	08:27:49	Airfield	Gulls	80	Runway	Loafing	Visual
10/2/2011	08:38:49	Airfield	Gulls	62	Runway	Loafing	Visual
10/2/2011	08:48:50	Airfield	Gulls	4	Sky	Overflight	Visual

In this scenario, a C-5 Galaxy departing Dover AFB is modeled with the assumption that the birds are distributed in the airfield environment according to a SPP and the speed of the birds is negligible compared to the speed of the aircraft (thus they are considered stationary). The encounter space $|A|$ is the volume of airspace around the aircraft equal to the radius of the wingspan swept through the distance the aircraft flies in the AO. This results in a disk of radius = 111 ft being swept through the air for the distance of the takeoff roll and for the distance required to climb out of the airport environment (in this case 500 ft of altitude). Table 5 contains the data for this scenario.

Airfield Environment Volume "AO"	Circle with diameter of 12903 ft (length of runway) Altitude = 500 ft (approx altitude when passing departure end of runway) (pi)(6451.5)(6451.5)(500)=65,346,308,032.5 ft cubed
C-5 Wingspan	222 ft
Takeoff Roll	8400 ft
Takeoff Roll "Volume"	(pi)(111)(111)(8400)=324,948,696 ft cubed
Climb Rate / Climb Distance	1800 ft/sec/4527 ft
Climb "Volume"	(pi)(111)(111)(4527)=175,140,304 ft cubed
λ	162/65346308033 = 2.4791E-09
|A|	324,978,696 + 175,140,304.4 = 500119000 ft cubed

Table 5 SPP in 3 Space Example Data

Figure 5 is a graphical depiction of the scenario:

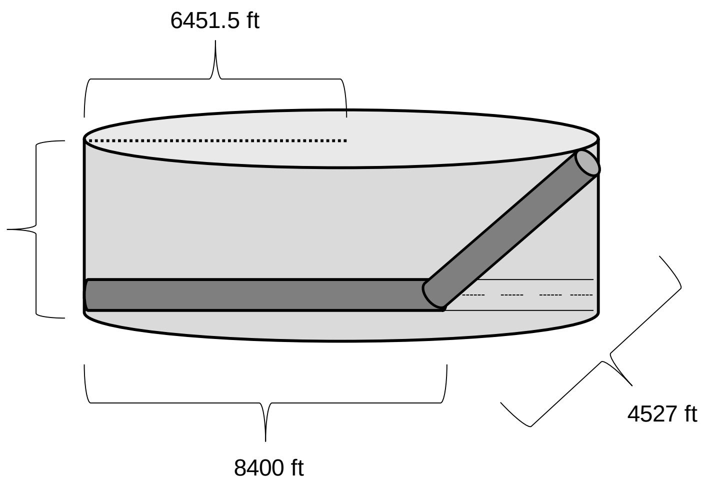


Figure 5 SPP in 3 Space Depiction

Using Equation 5 with the data from Table 5, the resulting probability of no (zero) aircraft / bird encounters is ~28.94%. The probability of one (1) encounter is ~35.88%. The probability of one or more strikes is 1 – 0.2894 = 0.7106 or ~ 71%. By using the disc of radius 111 ft (the wingspan), this model overestimated the area being swept through the AO.

This example assumed that the relative velocity between bird and aircraft as negligible and it assumed that the birds were stationary during the encounter. A commonly held belief (aircrew legend) is that birds will dive if they encounter an aircraft. However, research has shown that the most important feature of the execution of last-minute avoidance maneuvers by birds has been shown to be their unpredictability. Avoidance of aircraft in slow, straight and level flight seems to present little difficulty but at the speed and low altitudes at which birds are most likely to be encountered, aircraft are usually climbing or descending. This is where the unpredictability comes in. Observational evidence suggests that the tendency to attempt to dive or free fall beneath an aircraft rather than climb above it, which is more marked for some species than others, is not reliable enough to support a corresponding avoidance-climb response. Such a response might still lead to a strike, while the aircraft would be either at an increased rate of climb or in the transition from descent to climb, and aircraft performance margins would be reduced with a greater risk of significant consequences arising from strikes. Part of the underlying explanation for unpredictable aircraft avoidance is that birds do not always seem able to perceive aircraft as being in motion. Exceptionally, some hawks and eagles have been seen to 'attack' an aircraft which they encounter (SKYbrary 2012). Therefore, continue to assume the stationary location of birds with respect to aircraft for modeling.

2.7. Related Random Collision Models for Avians

Several researchers have studied the expected numbers of individual prey captured (# of birds ingested into an aircraft engine) from flocks of birds (Major, Dill and Eaves 1986). Their study determined the expected number of collisions for various predator speeds and trajectories, flock-predator initial distances and angles, and flock sizes, shapes, densities, trajectories, and speeds. Generally, larger predators and clustered predators (bigger engines in a triangular vice straight line configuration) caught more prey. A moving three-dimensional Poisson model was chosen to describe the interactions between predators (sets of engine frontal areas) and prey (dunlin flocks). The model, incorporated into an interactive computer program, calculated the expected number of captures of birds by each predator for each operator-supplied set of predator-prey trajectories and speeds. The model was also used to calculate, for individual predators, the exponential distribution of the inter-bird capture interval, its expected value, and its variance (Major, Dill and Eaves 1986).

Another area of study with respect to random collisions of birds and objects in motion is wind turbine rotors. When a bird flies through the disc swept out by blades of a wind turbine rotor, the probability of collision depends on the motions and dimensions of the bird and the blades. V.A. Tucker created a mathematical model that predicts the probability of a collision when a bird flies through the disk swept out by the rotor blades (Tucker 1996).

2.8. Extended Spatial Poisson Process

Extended Poisson process models derive their name from the fact that they are based upon generalizing the simplest Markov birth process, the Poisson process (T. Holzmann 2009). They involve representing a discrete distribution as the distribution of the number of events occurring in a finite time interval of a state-dependent Markov birth-death process (Podlich, Faddy and Smyth 1999) & (Faddy 1997).

Holzmann and Cochran furthered Faddy's (1990 & 2008) study of spatial data with respect to Extended SPP. They utilized a linearly decreasing arrival rate for the birth-death process of the Markov transition in order to "remove" encounter-able elements from the space (Holzmann and Cochran 2012). This resulted in $\lambda' = -(N-n)\ln(1-p)$ where p is the density function (calculated by $\frac{1}{|V|}$, where |V| is the "size" of the AO) and N is the number of encounter-able entities in the AO. This leads to the Markov transition matrix Q given by:

$$Q = \begin{bmatrix} -N\lambda & N\lambda & 0 & \cdots & 0 \\ 0 & -(N-1)\lambda & (N-1)\lambda & \ddots & 0 \\ 0 & 0 & -(N-2)\lambda & \ddots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (7)$$

where $\lambda = \ln\left(1 - \frac{1}{|V|}\right)$

This then leads to the probability distribution for the number of encounters in the AO for the specific window |A| as:

$$p = p_0 \exp\left(Q * |A|\right)$$

Where: $p_0 = \{1, 0, 0, \cdots, 0\}$ is the initial condition of no (0) encounters and the probability p_i of encounters at a count size of i (i = 0, 1,…,N) is the $(i + 1)^{\text{th}}$ element of p.

Example:

Given a space W of size |V| with 5 entities distributed throughout it, an object A passes through space W sweeping a path of size |A|.

If:

|V| = 10000 units cubed
|A| = 1500 units cubed
N = 5

The following is the transition matrix:

$$P_{ij} = \begin{bmatrix} 0.095 & 0.284 & 0.346 & 0.205 & 0.063 & 0.007 \\ 0 & 0.152 & 0.366 & 0.330 & 0.132 & 0.020 \\ 0 & 0 & 0.243 & 0.439 & 0.264 & 0.053 \\ 0 & 0 & 0 & 0.390 & 0.469 & 0.141 \\ 0 & 0 & 0 & 0 & 0.624 & 0.376 \\ 0 & 0 & 0 & 0 & 0 & 1.000 \end{bmatrix} \quad (8)$$

And

$$p = \begin{bmatrix} 0.095 & 0.284 & 0.346 & 0.205 & 0.063 & 0.007 \end{bmatrix}$$

Therefore, the probability of zero (0) encounters of object A with an entity is ~9.5%, and the probability of one (1) encounter is ~28.4%

However, if one were able to detect and "remove" an entity from the encounter-able list, then the initial condition could be set at $p_1 = \{0, 1, 0, 0, 0, 0]$ and row 2 of Equation 8 would be used. The Extended Spatial Poisson Process has application in the avian encounter models when calculating a risk score because it can "remove" an entity that has already been encountered and therefore can eliminate double counting and have a more realistic model by assuming that a hit bird is removed.

2.9. Matrix Exponentiation

The calculation of the matrix exponential, or an approximation of p, is crucial to the Extended Spatial Poisson Process. However, calculating the matrix exponential is not straightforward. The most commonly used algorithm to calculate the exponential is that of Golub and Van Loan (Faddy and Smith, Extended Poisson process modeling of dilution series data 2008). Current applications, such as ®MATLAB and ®SAS quickly solve matrix exponentiation of reasonable size. However, these programs are not readily available. Appendix II contains VBA code that allows the user to obtain p by an iterative method of Taylor expansions:

$$e^{Q*|A|} = \sum_{k=0}^{\infty} \frac{(Q*|A|)^k}{k!} \tag{9}$$

Additionally, it also worth noting that $Q * |A|$ (referred to a Q from here on out) has no repeated eigenvalues (the diagonal elements are unique) and Q is upper triangular. Given this information, Q can be definitely be diagonalized.

If the eivenvalues of Q are $\lambda_1, \lambda_2, \ldots, \lambda_n$, then each eigenvector ($x_1, x_2, \ldots, x_n$) of Q is linearly independent. If these eigenvalues are the columns of matrix S, then S^{-1} Q S is the diagonal matrix Λ and the eigenvalues of Q are the diagonal elements of Λ. This result in the following diagonal matrix: (Strang 2006).

$$Q = S\Lambda S^{-1} \rightarrow S^{-1}QS = \Lambda = \begin{bmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_n \end{bmatrix} \tag{10}$$

Additionally, it follows that:

$$e^{Q} = S \cdot e^{Q} \cdot S^{-1} = S \begin{bmatrix} e^{\lambda_1} & & \\ & \ddots & \\ & & e^{\lambda_n} \end{bmatrix} S^{-1} \tag{11}$$

Therefore, the eigenvectors associated with the eigenvalues of Q can be determined and Q can be exponentiated as well.

3. Methodology

3.1. Overview

This chapter describes the formulation and use of the model. The Spatial Poisson Process, using specified parameters, will be used to determine the probability of an aircraft / bird encounter. This probability will then be aggregated within the specific bird species and then multiplied by a scaled risk score and then all species will be summed in order to give the decision maker an additional analysis tool when planning missions.

3.2. Assumptions

The following assumptions are made with respect to model formulation and use:

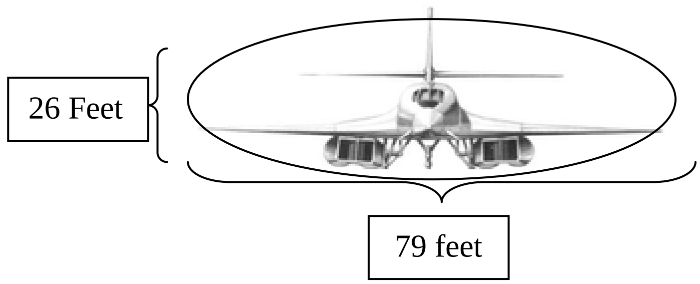
- The baseline for an Operations Area (AO) is 1km x 1km x 3500 feet
- Estimated number and type of birds in local area are known to an acceptable level
- The velocity of birds in the AO is negligible compared to the speed of the aircraft and can be considered stationary
- The number of birds in the AO is constant
- An aircraft/bird encounter within the ellipse formed by the height and width of an aircraft will be considered a strike
- Altitude distributions of birds (if known) will following a gamma distribution (birds within an altitude band will be distributed via a Poisson process)
- SBE's, as outlined in the independence section, are utilized as needed to determine species and numbers of birds in the AO.

3.3. Methods

The Poisson spatial srocess in 3-space will be utilized to determine the probability of a bird strike. Recall Equation 5:

$$p_n = P(\omega \in \Omega^X : \omega(A) = n) = \frac{(\lambda |A|)^n}{n!} e^{-\lambda |A|}$$

|A| is the volume of space defined by sweeping the elliptical disc around an aircraft through the AO for a defined distance *d*. The ellipse around an aircraft is a greater area than the frontal area of an aircraft and can be substituted for actual frontal area in order to provide a more precise calculation. The following is an illustration of the ellipse.



λ is the intensity function = # birds in the AO / Total volume of AO

Lambda will be scaled by the following parameters:

- gamma height distribution of the specific bird species
- the percentage of a specific bird species to the total number in the AO.
- "n" is the number of birds in the AO

The Dolbeer risk score ranking (Table 2) will be used for the risk each species presents. This data is scaled using Equation 12 (removing deer and coyote) and the result is the multiplier for which the "probability of bird" strike will be multiplied against in order to determine a risk score.

Species	Relative Hazard Score
Vultures	63
Geese	52
Cranes	48
Osprey	50
Pelicans	44
Ducks	37
Hawks	25
Eagles	31
Rock Dove	24
Gulls	22
Herons	22
Mourning Dove	17
Owls	16
American Kestrel	14
Shorebirds	12
Crows - Ravens	12
Blackbirds - Starling	9
Sparrows	4
Swallows	2

Figure 6 Risk Score Multiplier

The methods used and the parameters needed result in three cases for the planners. The baseline inputs for all three cases consist of complete AO data and aircraft data. The following are the specific case inputs and outputs.

Case #1 inputs are total birds in the AO. Case #1 outputs are the probabilities of an encounter.

Case #2 inputs are total birds in the AO and the specific species percentages in the AO. This results in outputs of the probabilities of an encounter and a risk score / risk matrix.

Case #3 inputs are total birds in the AO and the specific species percentages in the AO, gamma height distribution and the aircraft altitude band to be flown. The outputs for case #3 are the probabilities of an encounter and a risk score / risk matrix, with a higher fidelity due to the addition of the gamma distribution and aircraft altitude band.

3.4. Parameters and Calculation

The following are user inputs:

Identifier	Description	Used in
L	AO Length in meters	$\|V\|$ → AO Volume
W	AO Width in meters	$\|V\|$ → AO Volume
H	AO Height in meters	$\|V\|$ → AO Volume
A_Mx	Aircraft Altitude Max	$\|\nu\|$ → Modified AO Volume
A_Mn	Aircraft Altitude Min	$\|\nu\|$ → Modified AO Volume
A_W	Aircraft Wingspan	$\|A\|$ → Aircraft Volume
A_H	Aircraft Height	$\|A\|$ → Aircraft Volume
Surface_Area	Aircraft Frontal SA	$\|A\|$ → Aircraft Volume
A_D	Aircraft Distance	$\|A\|$ → Aircraft Volume
T	Total # birds in AO	λ → Intensity Function
$B_{species}\alpha$	Gamma Alpha value	$\lambda'_{species}Alt$ → Intensity Function modified volume
$B_{species}\beta$	Gamma Beta value	$\lambda'_{species}Alt$ → Intensity Function modified volume
$B_{species}T$	% of T	$\lambda'_{species}$ → Intensity Function bird % known
$RH_{species}$	Species Risk Score	$R_{species}$ = total risk by species

The following calculations are needed for model implementation:

$|V| = L * W * H$ {length * width * height}
$|\nu| = L * W * (A\_Mx - A\_Mn)$ {length * width * altitude band height}
$|A| = ½(A\_W) * ½(A\_H) * \pi * (A\_D)$ {area of ellipse * distance}

or

$|A|' = Surface\_Area*(A\_D)$ {Aircraft Frontal Surface Area * distance}
$\lambda = T / |V|$
$\lambda'_{species} = (B_{species}T * T) / |V|$
$\lambda'_{species}Alt = [(B_{species}T * T)*(GammaCDF(A\_Mx) – GammaCDF(A\_Mn)] / |\nu|$

3.5. Flow Diagram

The following flow diagram illustrates the inputs, decisions, calculations and outputs for the model.

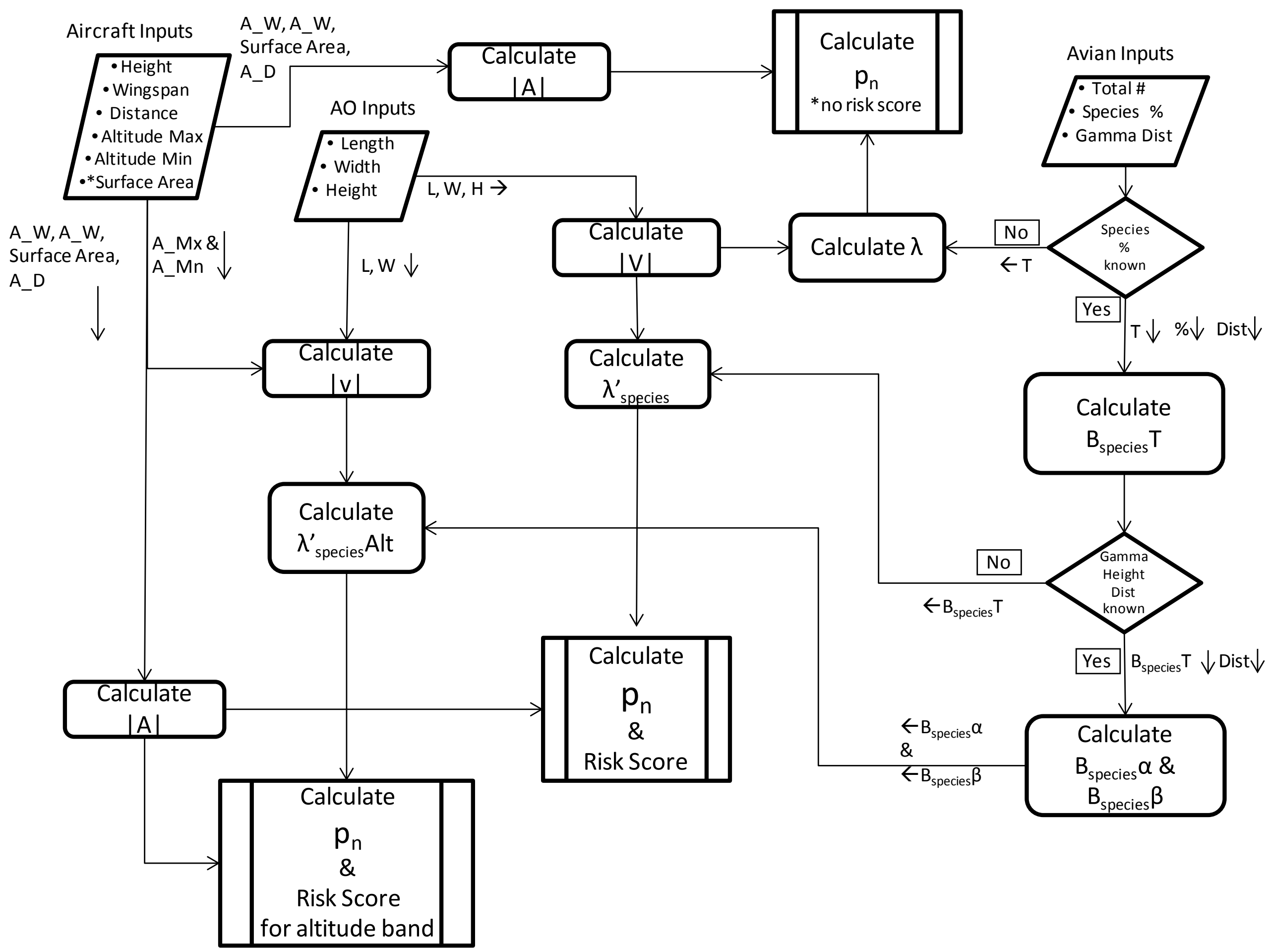


Figure 7 Model Flow Chart

3.6. Model

This section describes the model formulation. There are three different models that can be used to determine a bird strike and an associated risk score, based upon the number and type of known parameter inputs. These models use the spatial Poisson process to determine a strike probability. The strike probability is used to determine the expected number of hits for a flight through an AO for a specific species of bird. The expected number of hits (subtracting double counting / intersections) per specific species is then multiplied by the risk multiplier to get a risk score. These individual risk scores are then aggregated over all the species to get a total risk score. Figure 7 depicts the interaction of inputs, calculations and outputs for the models.

The change in the input parameters affects the λ used for the probability function and thus the strike probability and the risk score. The following table outlines the inputs and outputs for the three cases.

Table 6 Model input/output

	INPUT							**OUTPUT**	
Case #	AO Volume	Aircraft	Distance Through AO	Aircraft Altitude Band	Total # Birds in AO	Specific Species %	Avian	Strike Probability	Risk Score
1	X	X	X		X			X	
2	X	X	X		X	X		X	X
3	X	X	X	X	X	X	X	X	X

The following equations are used for the three cases outlined in Table 6 and form the basis for the model. These equations are embedded in the decision support system in order to generate strike probability and risk score.

Case # 1: General encounter model:

The probability of an encounter for all the birds in the AO is calculated using the spatial Poisson process:

$$p_n = e^{-\lambda * |A|} * \frac{(\lambda * |A|)^n}{n!}$$

This case gives the probability of an encounter with only the basic parameters of AO size, aircraft size (surface area or wingspan/height), aircraft distance, and total number of birds known. There is no risk score associated with this case because specific species percentages are not known.

Case #2: General encounter model with risk score added:

The probability of an encounter for a specific species is calculated using the Spatial Poisson Process with a modified intensity parameter $\lambda'_{species}$ (generated using percentage of specific bird in the AO).

$$p_{species}(n) = e^{-\lambda'_{species} * |A|} * \frac{\left(\lambda'_{species} * |A|\right)^n}{n!}$$

The risk score (by species) is calculated by aggregating over the specific species (the number of birds x probability of hit (eliminating double counts)) x species specific risk level:

$$R_{species} = \sum_{n=0}^{N} n * p_{species}(n) * RH_{species}$$

Total Risk Score is calculated by summed over all the species.

$$R = \sum_{\text{All species}} R_{species}$$

This model gives the probability of an encounter for each specific species and a risk level score.

Case # 3: General encounter model with risk score for a given altitude band:

This case adds another element, the gamma height distribution, to the problem set. By using the gamma distribution in the intensity parameter ($\lambda'_{species}$) function, the intensity parameter now becomes itself a random variable. This is a doubly-stochastic process, where the intensity parameter ($\lambda'_{species}$) is now also stochastic process. This is also called a Cox process (Cox 1955).

The probability of encounter within the altitude band is determined using a doubly-stochastic process modified intensity parameter $\lambda'_{species}Alt$:

$$p_{species}Alt(n) = e^{-\lambda'_{species}Alt*|A|} * \frac{(\lambda'_{species}Alt * |A|)^n}{n!}$$

The risk score (by species) is calculated by aggregating over the specific species (the number of birds x probability of hit (eliminating double counts)) x species specific risk level:

$$R_{species}Alt = \sum_{n=0}^{N} n * p_{species}Alt(n) * RH_{species}$$

Total Risk Score is calculated by summing over all the species.

$$R = \sum_{\text{All species}} R_{species}\,Alt$$

This model calculates the probability of an encounter (eliminating double counts) and the risk score for a particular band of altitude within the overall AO. The model requires a gamma distribution for the specific species that are in the AO and the aircraft altitude band to be flown in order to calculate these values. Figure 8 is a visual representation of the case #3.

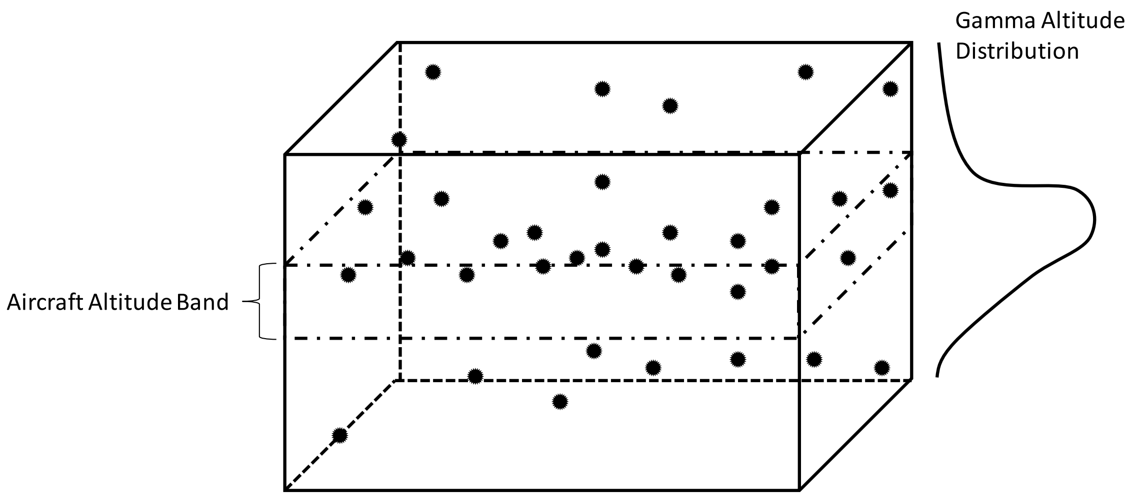


Figure 8 Gamma Altitude Distribution and Altitude Band

In order to give a more precise estimate and to eliminate entities that have already been encountered, one can apply the extended spatial Poisson process to the encounter probability when determining the risk score. Therefore, the model determines:

P(x+1 | x , where x = 1, 2, …, N-1 & N = total # birds in AO).

Figure 9 and Figure 10 are the outputs from the spatial Poisson process and the extended spatial Poisson process respectively for the same scenario. In this scenario, the risk score = 1 for brevity.

n	P(n)	Risk
0	0.675232	0
1	0.265163	0.26516
2	0.052065	0.07652
3	0.006815	0.01424
4	0.000669	0.00185
5	5.25E-05	0.00018

Figure 9 Spatial Poisson Process Probability Output

Using the spatial Poisson process within the model and eliminating the double counts / intersections using $[\prod_{x=1}^{n-1}(1-p_x)$]* p_n * n and then aggregating over all "n" the Risk Score = .3580

Figure 10 contains the output when using the extended spatial Poisson process.

$$
\begin{array}{ccc}
 & & \text{To} \\
 & & \begin{array}{cccccc} 0 & 1 & 2 & 3 & 4 & 5 \end{array} \\
\text{From} & \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix}
0.675366 & 0.275763 & 0.04504 & 0.003678 & 0.00015 & 2.45288E-06 \\
0 & 0.730519 & 0.238627 & 0.029231 & 0.001591 & 3.24894E-05 \\
0 & 0 & 0.790176 & 0.193585 & 0.015809 & 0.000430334 \\
0 & 0 & 0 & 0.854704 & 0.139596 & 0.005699944 \\
0 & 0 & 0 & 0 & 0.924502 & 0.075497976 \\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\end{array}
$$

Figure 10 Extended Spatial Poisson Process

Using the extended spatial Poisson process and subtracting out the double counts and then calculating the Risk Score results in:

0.2757 +	{1 hit | 0 hit → from 0 to 1}
0.2757 * 0.2386 +	{2 hit | 1 hit → from 1 to 2}
0.2757 * 0.2386 * 0.1935 +	{3 hit | 2 hit → from 2 to 3}
0.2757 * 0.2386 * 0.1935 * 0.1395 +	{4 hit | 3 hit → from 3 to 4}
0.2757 * 0.2386 * 0.1935 * 0.1395 * 0.0754	{5 hit | 4 hit → from 4 to 5}
= 0.356	

Thus it is clear that the risk score using the extended spatial Poisson process is tighter because the model eliminated any entity that was removed when determining the next probability.

3.7. Universal Risk Score

In order to standardize the risk score, the model generates a scaled risk score. Using the relative risk score from Table 3 and removing deer and coyote, the relative risk score can be rescaled using Equation 12.

$$f(x) = \frac{(b-a)(x-\min)}{\max-\min} + a \tag{12}$$

where *b* = Rescaled Max; *a* = Rescaled Min; *min* = Original Min; *max* = Original Max

This paper uses the following factors for the rescale:

b = 10 and *a* = 0
min = risk score for entire AO if all birds are Swallows
max = risk score for entire AO if all birds are Vultures

This score is then used to generate 25/50/25 splits to highlight the top 25% in terms of risk score, the middle 50% and the lowest 25%. Figure 11 illustrates this sliding scale and how a specific scenario risk score relates to the maximum, 75% and 25% thresholds.

Figure 11 Risk Score Scale

Additionally, using a Risk Filtering and Ranking Method (RFRM) that compares the effect (total risk score) and the likelihood of an event happening (1-P(0)), the model generates a graphic that depicts where the scenario's risk level is located on the graph using a scale of low(L)/moderate(M)/high(H)/extremely high(H+). This chart can be tailored for different components or different airframes, depending on what is being measured.

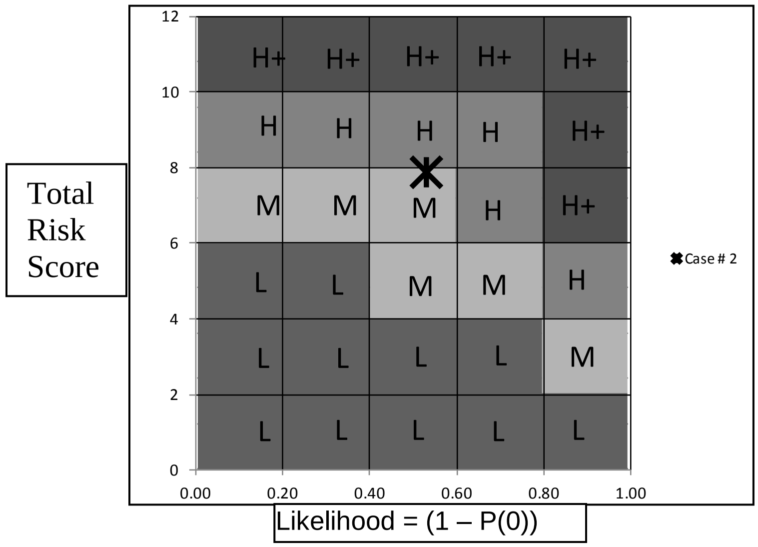


Figure 12 Risk Filtering and Ranking Method

3.8. Case Study

The following scenario represents a hypothetical flight through an operations area. The probability of hitting a bird is calculated and a risk score is generated.

Consider a low level bombing range of dimension 1km x 1km x 3500 ft. A B-1B with wings swept will pass through the operations area. The wingspan of the B-1B is 79ft and its height is 26ft. The aircraft will travel a distance of 1km through the operations area. The planned altitude block is 1000 – 2000 feet above ground level. Avian radar has depicted 350 birds in the operations area. Through historical data, specific bird species, their percent of the total and their altitude distribution have been determined according to Table 7.

Table 7 Scenario Inputs

Total # birds in AO = 500			
Species	% of Total	Alpha "α"	Beta "β"
Vultures	0.1	8	50
Geese	0.2	6	80
Pelicans	0.11	10	43
Ducks	0.14	5	151
Eagles	0.13	4.8	153
Hawks	0.09	4.6	155
Gulls	0.23	4.2	159

The aircrew wishes to determine the probability of an aircraft strike and the associated risk score according to the model.

The parameters from the above scenario are entered into the model and calculations are accomplished utilizing $\lambda'_{species}$Alt, |A|, and |v| for each species. The following figures (Figure 13) highlight the calculations that are being accomplished in support of the model for vultures and geese.

Vultures	Risk Sum	1.67569		
10	Max Risk	20.128		
Gamma Alpha "α" =	8			
Gamma Beta "β" =	50			
Gamma Altitude Band %	0.650963			
Total of specific species in AO	50			
λ'	1.07E-07			
Risk Multiplier	63			
			MAX	
n	P(n)	Risk	P(n)	Risk
0	0.84185	0	0.47	0
1	0.144928	1.44398	0.36	11.87
2	0.012475	0.21256	0.13	5.793829
3	0.000716	0.01807	0.03	1.897549
4	3.08E-05	0.00104	0.01	0.462387
5	1.06E-06	4.5E-05	0.00	0.086907
6	3.04E-08	1.5E-06	0.00	0.013139
7	7.49E-10	4.4E-08	0.00	0.001657
8	1.61E-11	1.1E-09	0.00	0.000179
9	3.08E-13	2.3E-11	0.00	1.69E-05
10	5.3E-15	4.5E-13	0.00	1.42E-06
11	8.3E-17	7.7E-15	0.00	1.08E-07
12	1.19E-18	1.2E-16	0.00	7.41E-09
13	1.58E-20	1.7E-18	0.00	4.67E-10
14	1.94E-22	2.3E-20	0.00	2.72E-11
15	2.23E-24	2.8E-22	0.00	1.47E-12
16	2.39E-26	3.2E-24	0.00	7.42E-14
17	2.43E-28	3.5E-26	0.00	3.51E-15
18	2.32E-30	3.5E-28	0.00	1.56E-16
19	2.1E-32	3.4E-30	0.00	6.56E-18
20	1.81E-34	3E-32	0.00	2.61E-19

Geese	Risk Sum	4.032935		
11	Max Risk	16.61358		
Gamma Alpha "α" =	6			
Gamma Beta "β" =	80			
Gamma Altitude Band %	0.58619437			
Total of specific species in AO	100			
λ'	1.92195E-07			
Risk Multiplier	52			
			MAX	
n	P(n)	Risk	P(n)	Risk
0	0.733410081	0	0.47	0
1	0.227393999	3.152289	0.36	9.80
2	0.035251786	0.755121	0.13	4.782208
3	0.003643275	0.112936	0.03	1.566231
4	0.0002824	0.011629	0.01	0.381653
5	1.75116E-05	0.000901	0.00	0.071733
6	9.04914E-07	5.59E-05	0.00	0.010845
7	4.00812E-08	2.89E-06	0.00	0.001368
8	1.5534E-09	1.28E-07	0.00	0.000148
9	5.35147E-11	4.96E-09	0.00	1.4E-05
10	1.65922E-12	1.71E-10	0.00	1.18E-06
11	4.67675E-14	5.29E-12	0.00	8.89E-08
12	1.20836E-15	1.49E-13	0.00	6.12E-09
13	2.88193E-17	3.86E-15	0.00	3.86E-10
14	6.38246E-19	9.2E-17	0.00	2.25E-11
15	1.31926E-20	2.04E-18	0.00	1.21E-12
16	2.55647E-22	4.21E-20	0.00	6.12E-14
17	4.66256E-24	8.16E-22	0.00	2.9E-15
18	8.03127E-26	1.49E-23	0.00	1.29E-16
19	1.31058E-27	2.56E-25	0.00	5.42E-18
20	2.03172E-29	4.18E-27	0.00	2.16E-19

Figure 13 Species Specific Calculations

Given the parameters, the model follows Case #3 from Table 6 and the bird strike probabilities and an associated risk score can be computed. Figure 14 is the final model with all the inputs and the associated outputs.

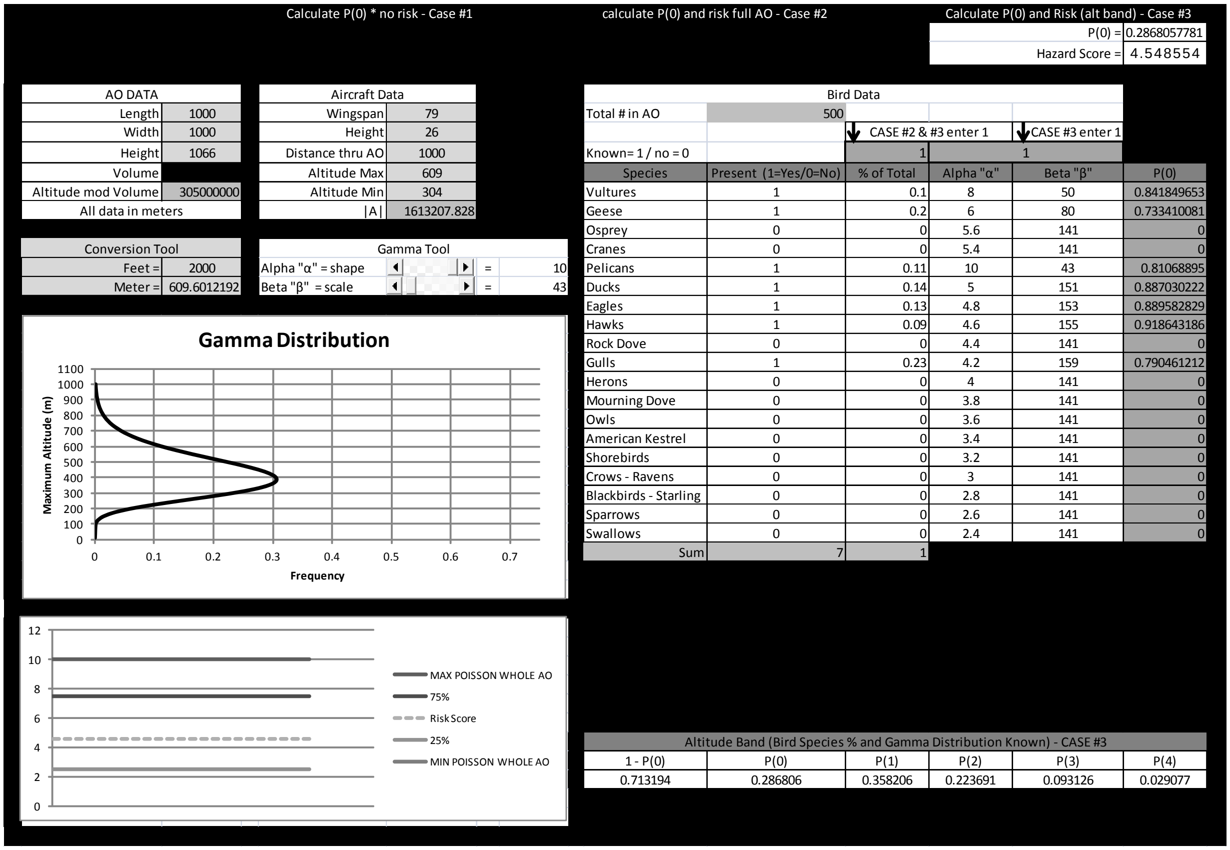


Figure 14 Aircraft / Avian Encounter Model

Note that *p(0)*, the probability of no bird strikes, is ~29% and the probability of at least one bird strike (1- *p(0)*) is ~71%. The risk score is 4.54 and falls within the middle 50% of the sliding scale risk score graphic.

Further risk management fidelity can be gained from Figure 15, the RFRM graph. The RFRM graph gives an overall factor of Moderate for this scenario.

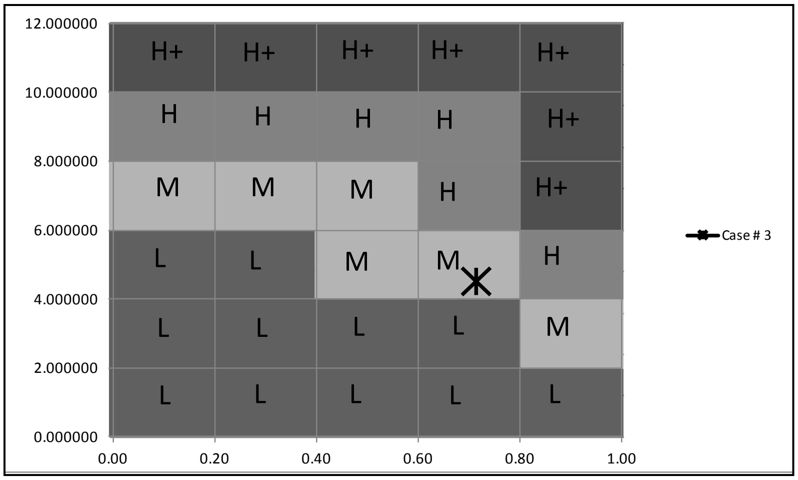


Figure 15 RFRM Graph

3.9. Summary

Using the spatial Poisson process to calculate a probability for an aircraft / avian encounter provides a closed form mathematical model that is fairly easy to compute. Assessing the risk is more difficult and requires additional parameter inputs and greater fidelity of those parameters. Additionally, there are several key assumptions that need to be made (independence of birds, small bird estimates, species percentage, etc) when computing a relative risk score. If these assumptions are met, then planners can use this model to underpin their decisions when assessing overall risk management. The extended spatial Poisson process is a very powerful method that provides a higher degree of accuracy when determining encounter probability as it allows for the removal of entities, which are already encountered, from the AO.

4. Analysis

4.1. Bird Density

The following scenario is for a B-1B transiting through an AO of 1km x 1km x 3500ft. The number of total birds for in the AO is the independent variable in the analysis and the total number varies from 0 to 3000 total birds. The dependent variable is the probability of one or more bird strikes (1-P(0)).

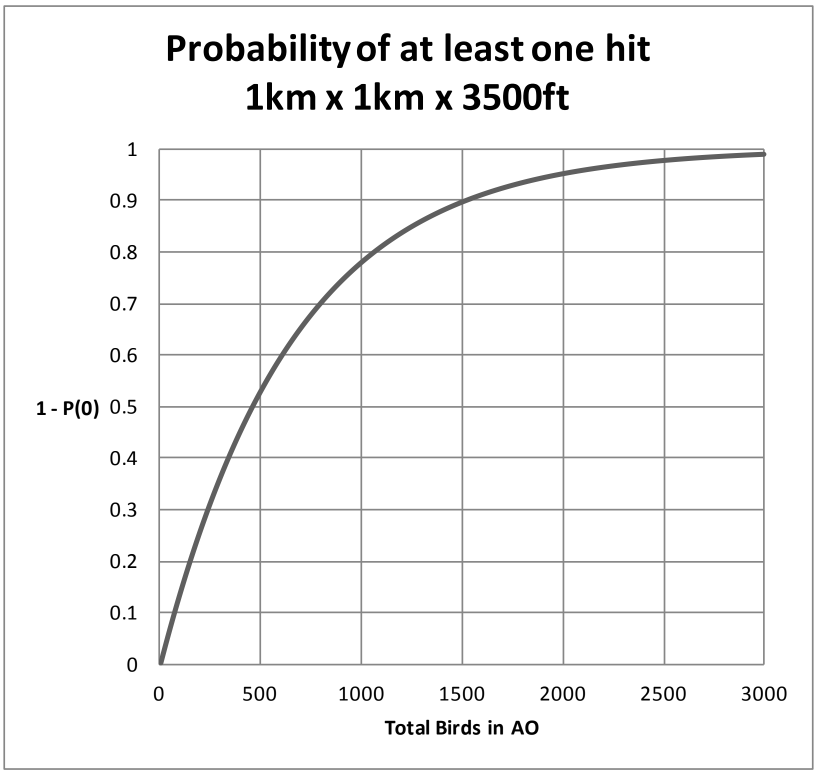


Figure 16 Sensitivity Analysis [1-P(0)]

As expected the probability of one of more strikes increases as the number of total birds in the AO increases. Figure 16 indicates that the probability of one or more bird strikes increases at a lognormal rate as the number of birds in the AO increases.

4.2. Species Percentage / Risk Score

The following scenario is for a B-1B transiting through an AO of 1km x 1km x 3500ft. In this analysis, there were 7 different species present in the AO. They consisted of vultures, geese, cranes, pelicans, ducks, eagles and hawks. These 7 species were identified by USAF as the 7 most damaging species for Air Combat Command (Kelly, et al. 1999). The independent variable in this analysis was the percentage (of the total # of birds) that were vultures. The other six species were held at equal percentages of the remainder (calculation: 1/7 * (1 – Vulture %)). The dependent variable is the Risk Score. The total number of birds in the AO was also varied (independently) from 100 to 500 by increments of 100. The risk score is scaled from 0 to 1 for brevity.

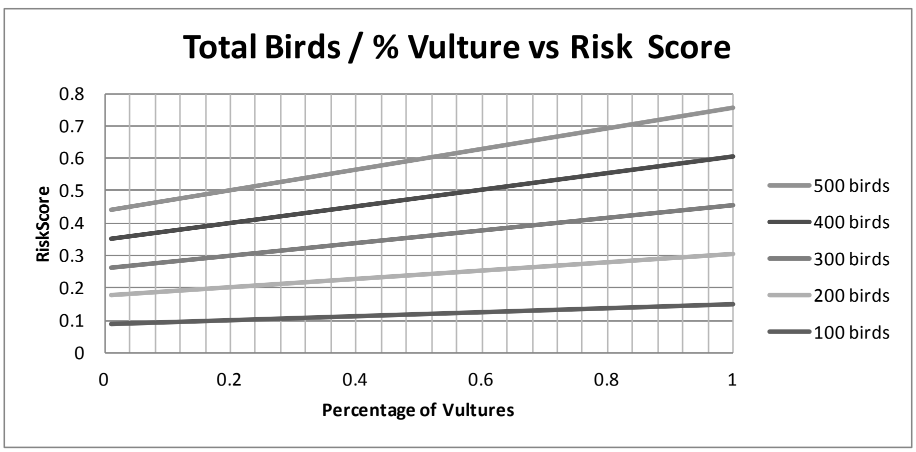
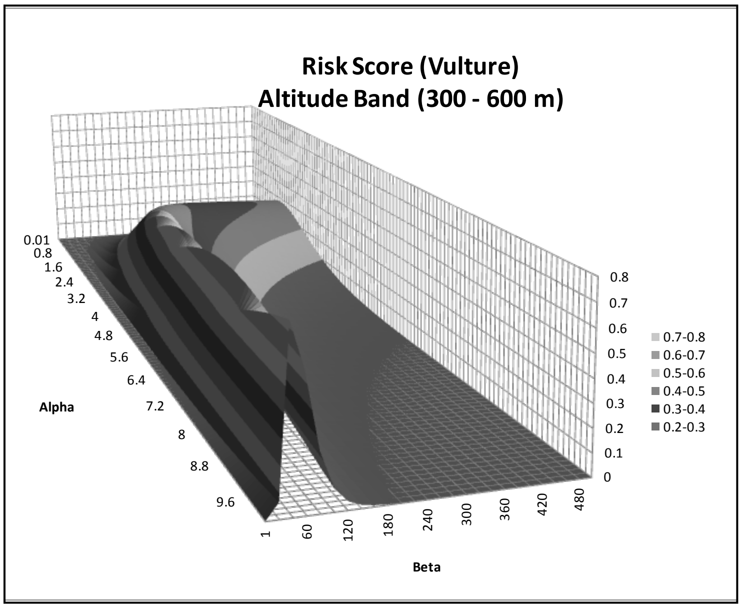


Figure 17 Sensitivity Analysis [Risk Score / Vulture probability]

Figure 17 highlights that as the slope of the line associated with the risk score is steeper as the total number of birds in the AO is increase. The risk score increases for all level of total number of birds as the percentage of vultures increases (as expected).

4.3. Gamma Altitude Band

The following scenario is for a B-1B transiting through an AO of 1km x 1km x 3500ft at an altitude of 300 to 600 meters. In this scenario, there are 200 vultures in the AO. The independent variables in this analysis are the alpha and beta values associated with the gamma altitude distribution. They are varied from .01 to 10 for the alpha value and from 1 to 500 for the beta value. The dependent variable is the risk score. The risk score is scaled from 0 to 1 for brevity.



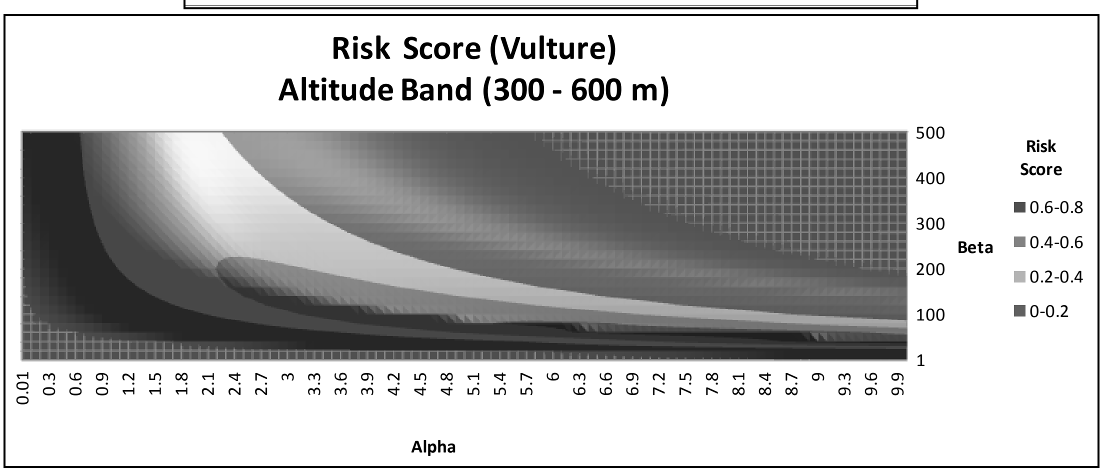


Figure 18 Sensitivity Analysis [Gamma Altitude Band]

From Figure 18 it can be seen that that the beta variable is the most influential (50 to 70) for alpha values between 4 – 10. The alpha value is the most influential when alpha values are below 4 and they are most influential from 1.2 - 2.4 for beta value from 100 to 500.

5. Conclusion

This paper combines the powerful methods of the spatial Poisson processes and the extended spatial Poisson processes with relative risk score values and with gamma altitude distributions. The spatial Poisson process accounts for the spatial distribution of birds within a bounded area of operations. The extended spatial Poisson process allows for over-dispersed and under-dispersed object densities in this bounded space with respect to the spatial Poisson process and provides for a more accurate estimation. The relative risk scores account for differing hazards associated with striking different species of birds. The gamma altitude distributions provides further fidelity when determining the hazard associated with a particular altitude band to be flown. The model allows for varying levels of user input and provides numerous outputs to include collision probabilities, a raw risk score, a standardized and graphical relative risk score, a gamma distribution determination tool and a risk filtering and ranking method graphic. It is recommended that leaders and resident BASH / Wildlife experts incorporate this model (or a locally adjusted model) into their decision making process.

Future research in the area of aircraft / avian encounters should account for the varying flock sizes (Hemelrijk and Hildenbrandt 2011). The Shot noise Cox processes and the finite Gibbs point process (Møller and Waagepeterson 2006) could be utilized to model some of the clustering of birds within an AO. Additionally, flock shapes and density distributions within the flock should be studied. This would allow for a greater fidelity on types of birds that are observed on a radar return.

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7. Appendix I - Model

The complete model allows the decision maker to input parameters for AO data, Aircraft data and Bird data. Any one of the three cases can be computed if the parameters of species percentage and gamma height distribution are known. Note that the aircraft data currently is utilizing an ellipse around the aircraft as the encounterable area. If the decision maker knows the exact frontal surface area or desires a specific surface area (e.g. engine intake), then the model can easily be modified to account for this. The gamma distribution graph allows the decision maker to match up the specific bird height parameters with what is known. The graph in the lower left corner is the sliding scale for the risk score. The risk score adjusts based upon total number and species of birds in the AO. The tables in the lower right are probabilities P(n) {n equal >= 1, and n = 0, 1, 2, 3 4}.

AVIAN ENCOUNTER / RISK MODEL

Calculate P(0) * no risk - Case #1	
AO P(0) =	0.469229128
AO (1-P(0)) =	0.5307708721

calculate P(0) and risk full AO - Case #2	
P(0) =	0.4692291279
Hazard Score =	4.548554

Calculate P(0) and Risk (alt band) - Case #3	
P(0) =	0.2868057781
Hazard Score =	4.548554

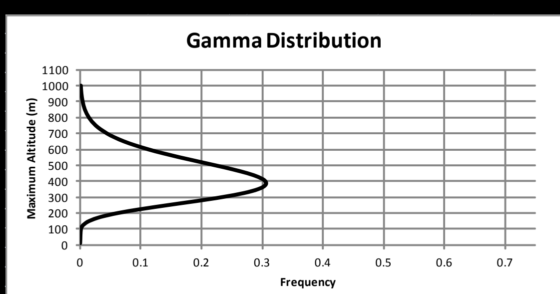
AO DATA	
Length	1000
Width	1000
Height	1066
Volume	
Altitude mod Volume	305000000
All data in meters	

Aircraft Data	
Wingspan	79
Height	26
Distance thru AO	1000
Altitude Max	609
Altitude Min	304
|A|	1613207.828

Conversion Tool	
Feet =	2000
Meter =	609.6012192

Gamma Tool		
Alpha "α" = shape	=	10
Beta "β" = scale	=	43

Bird Data					
Total # in AO	500				
		CASE #2 & #3 enter 1	CASE #3 enter 1		
Known= 1 / no = 0		1	1		
Species	Present (1=Yes/0=No)	% of Total	Alpha "α"	Beta "β"	P(0)
Vultures	1	0.1	8	50	0.841849653
Geese	1	0.2	6	80	0.733410081
Osprey	0	0	5.6	141	0
Cranes	0	0	5.4	141	0
Pelicans	1	0.11	10	43	0.81068895
Ducks	1	0.14	5	151	0.887030222
Eagles	1	0.13	4.8	153	0.889582829
Hawks	1	0.09	4.6	155	0.918643186
Rock Dove	0	0	4.4	141	0
Gulls	1	0.23	4.2	159	0.790461212
Herons	0	0	4	141	0
Mourning Dove	0	0	3.8	141	0
Owls	0	0	3.6	141	0
American Kestrel	0	0	3.4	141	0
Shorebirds	0	0	3.2	141	0
Crows - Ravens	0	0	3	141	0
Blackbirds - Starling	0	0	2.8	141	0
Sparrows	0	0	2.6	141	0
Swallows	0	0	2.4	141	0
Sum	7	1			



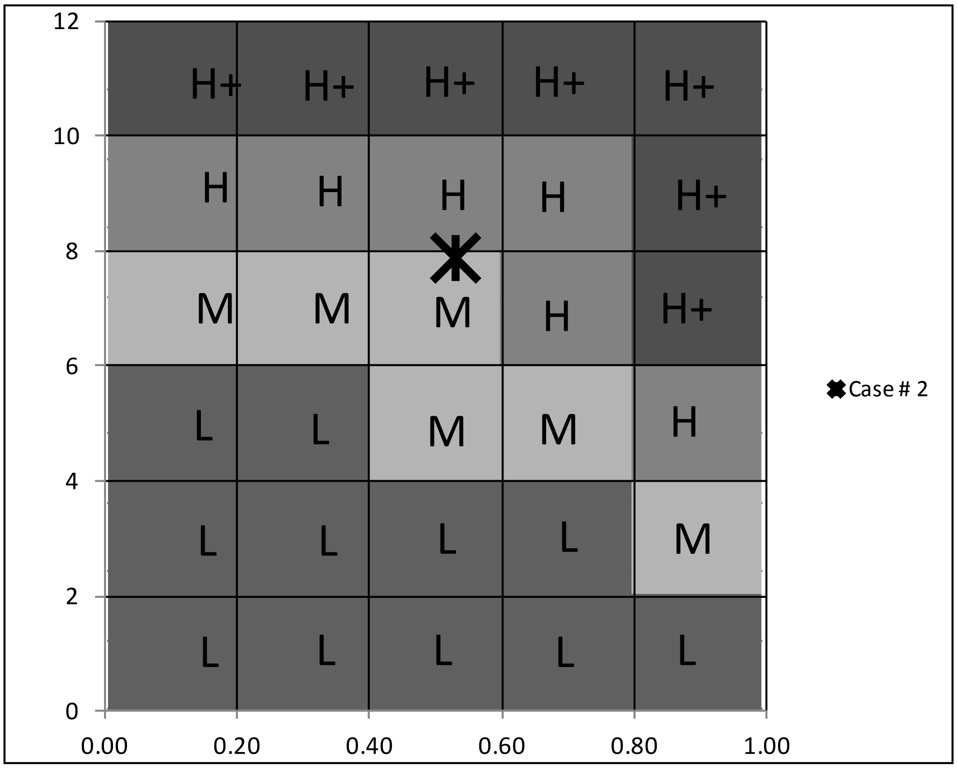
Full AO (Bird Species Not Known) - CASE #1					
1 - P(0)	P(0)	P(1)	P(2)	P(3)	P(4)
0.530770872	0.469229128	0.355048828	0.134326348	0.033879974	0.00640894
Full AO (Bird Species % Known) - CASE # 2					
1 - P(0)	P(0)	P(1)	P(2)	P(3)	P(4)
0.5307708721	0.4692291279	0.3550488284	0.1343263483	0.0338799744	0.0064089400
Altitude Band (Bird Species % and Gamma Distribution Known) - CASE #3					
1 - P(0)	P(0)	P(1)	P(2)	P(3)	P(4)
0.713194	0.286806	0.358206	0.223691	0.093126	0.029077

The Risk Filtering and Ranking Method is displayed via the following graphs where:

Y axis is the effect (damage) on an aircraft, increasing

from:Marginal→Moderate→Serious→Critical→Catastrophic:

X axis is the likelihood from probability of a hit = 0 to probability of a hit = 1



The calculations for each of the bird species are computed utilizing the following tables:

Vultures	Risk Sum	9.08936		
10	Max Risk	48.4163		
Gamma Alpha "α" =	8			
Gamma Beta "β" =	50			
Gamma Altitude Band %	0.650963			
Total of specific species in AO	130			
λ'	2.77E-07			
Risk Multiplier	63			
			MAX	
n	P(n)	Risk	P(n)	Risk
0	0.63916	0	0.22	0
1	0.286088	6.50361	0.33	16.37
2	0.064027	2.07821	0.25	16.51847
3	0.009553	0.43532	0.13	9.347712
4	0.001069	0.06433	0.05	4.115684
5	9.57E-05	0.00719	0.01	1.482173
6	7.14E-06	0.00064	0.00	0.44207
7	4.56E-07	4.8E-05	0.00	0.11109
8	2.55E-08	3.1E-06	0.00	0.023997
9	1.27E-09	1.7E-07	0.00	0.004539
10	5.69E-11	8.5E-09	0.00	0.000763
11	2.31E-12	3.8E-10	0.00	0.000115
12	8.63E-14	1.6E-11	0.00	1.59E-05
13	2.97E-15	5.8E-13	0.00	2E-06
14	9.5E-17	2E-14	0.00	2.33E-07
15	2.83E-18	6.4E-16	0.00	2.52E-08
16	7.93E-20	1.9E-17	0.00	2.54E-09
17	2.09E-21	5.3E-19	0.00	2.41E-10
18	5.19E-23	1.4E-20	0.00	2.14E-11
19	1.22E-24	3.5E-22	0.00	1.8E-12
20	2.74E-26	8.2E-24	0.00	1.43E-13

Geese	Risk Sum	14.33661		
11	Max Risk	39.96267		
Gamma Alpha "α" =	6			
Gamma Beta "β" =	80			
Gamma Altitude Band %	0.58619437			
Total of specific species in AO	220			
λ'	4.22829E-07			
Risk Multiplier	52			
			MAX	
n	P(n)	Risk	P(n)	Risk
0	0.505548849	0	0.22	0
1	0.344840234	8.866346	0.33	13.51
2	0.117609591	3.962294	0.25	13.63429
3	0.026740917	1.192428	0.13	7.715572
4	0.004560066	0.263873	0.05	3.397072
5	0.000622094	0.044792	0.01	1.223381
6	7.07228E-05	0.006107	0.00	0.364883
7	6.89154E-06	0.000694	0.00	0.091693
8	5.87599E-07	6.76E-05	0.00	0.019807
9	4.45342E-08	5.77E-06	0.00	0.003746
10	3.03772E-09	4.37E-07	0.00	0.00063
11	1.88369E-10	2.98E-08	0.00	9.53E-05
12	1.07074E-11	1.85E-09	0.00	1.31E-05
13	5.61818E-13	1.05E-10	0.00	1.65E-06
14	2.7373E-14	5.51E-12	0.00	1.93E-07
15	1.24476E-15	2.69E-13	0.00	2.08E-08
16	5.30665E-17	1.22E-14	0.00	2.1E-09
17	2.12925E-18	5.21E-16	0.00	1.99E-10
18	8.06879E-20	2.09E-17	0.00	1.77E-11
19	2.89674E-21	7.92E-19	0.00	1.49E-12
20	9.87949E-23	2.84E-20	0.00	1.18E-13

Osprey	Risk Sum	9.746177		
12	Max Risk	38.42565		
Gamma Alpha "α" =	5.6			
Gamma Beta "β" =	141			
Gamma Altitude Band %	0.292257901			
Total of specific species in AO	350			
λ'	3.35378E-07			
Risk Multiplier	50			
			MAX	
n	P(n)	Risk	P(n)	Risk
0	0.582145837	0	0.22	0
1	0.314960856	6.580385	0.33	12.99
2	0.085202311	2.438886	0.25	13.1099
3	0.01536579	0.603547	0.13	7.418819
4	0.002078355	0.107174	0.05	3.266416
5	0.000224892	0.014466	0.01	1.176328
6	2.02791E-05	0.001565	0.00	0.350849
7	1.56738E-06	0.000141	0.00	0.088167
8	1.06001E-07	1.09E-05	0.00	0.019046
9	6.37224E-09	7.38E-07	0.00	0.003602
10	3.4476E-10	4.43E-08	0.00	0.000606
11	1.6957E-11	2.4E-09	0.00	9.17E-05
12	7.64526E-13	1.18E-10	0.00	1.26E-05
13	3.18181E-14	5.32E-12	0.00	1.59E-06
14	1.22962E-15	2.21E-13	0.00	1.85E-07
15	4.43511E-17	8.56E-15	0.00	2E-08
16	1.49972E-18	3.09E-16	0.00	2.02E-09
17	4.77293E-20	1.04E-17	0.00	1.91E-10
18	1.43462E-21	3.32E-19	0.00	1.7E-11
19	4.08515E-23	9.98E-21	0.00	1.43E-12
20	1.1051E-24	2.84E-22	0.00	1.14E-13

Cranes	Risk Sum	20.05199		
13	Max Risk	36.88862		
Gamma Alpha "α" =	5.4			
Gamma Beta "β" =	141			
Gamma Altitude Band %	0.317030651			
Total of specific species in AO	550			
λ'	5.71695E-07			
Risk Multiplier	48			
			MAX	
n	P(n)	Risk	P(n)	Risk
0	0.397618518	0	0.22	0
1	0.366708541	10.60312	0.33	12.47
2	0.169100719	6.19287	0.25	12.5855
3	0.051985069	2.37282	0.13	7.122066
4	0.011985966	0.691533	0.05	3.135759
5	0.002210841	0.157533	0.01	1.129275
6	0.000339829	0.028993	0.00	0.336815
7	4.47731E-05	0.004455	0.00	0.08464
8	5.16157E-06	0.000587	0.00	0.018284
9	5.28924E-07	6.77E-05	0.00	0.003458
10	4.87807E-08	6.93E-06	0.00	0.000581
11	4.08987E-09	6.39E-07	0.00	8.8E-05
12	3.14328E-10	5.36E-08	0.00	1.21E-05
13	2.22994E-11	4.12E-09	0.00	1.53E-06
14	1.46899E-12	2.92E-10	0.00	1.78E-07
15	9.03199E-14	1.93E-11	0.00	1.92E-08
16	5.20616E-15	1.18E-12	0.00	1.94E-09
17	2.82438E-16	6.82E-14	0.00	1.83E-10
18	1.44712E-17	3.7E-15	0.00	1.63E-11
19	7.02435E-19	1.9E-16	0.00	1.37E-12
20	3.23915E-20	9.21E-18	0.00	1.09E-13

Pelican	Risk Sum	16.21572		
14	Max Risk	33.81457		
Gamma Alpha "α" =	10			
Gamma Beta "β" =	43			
Gamma Altitude Band %	0.72143785			
Total of specific species in AO	220			
λ'	5.20381E-07			
Risk Multiplier	44			
			MAX	
n	P(n)	Risk	P(n)	Risk
0	0.431933625	0	0.22	0
1	0.362601086	9.063185	0.33	11.43
2	0.152198787	4.849582	0.25	11.53671
3	0.042589449	1.72576	0.13	6.528561
4	0.008938283	0.462348	0.05	2.874446
5	0.001500708	0.096166	0.01	1.035169
6	0.00020997	0.016122	0.00	0.308747
7	2.51809E-05	0.002255	0.00	0.077587
8	2.64237E-06	0.00027	0.00	0.01676
9	2.46469E-07	2.84E-05	0.00	0.00317
10	2.06907E-08	2.65E-06	0.00	0.000533
11	1.57904E-09	2.22E-07	0.00	8.07E-05
12	1.10465E-10	1.7E-08	0.00	1.11E-05
13	7.13335E-12	1.19E-09	0.00	1.4E-06
14	4.27738E-13	7.66E-11	0.00	1.63E-07
15	2.39386E-14	4.59E-12	0.00	1.76E-08
16	1.256E-15	2.57E-13	0.00	1.78E-09
17	6.20232E-17	1.35E-14	0.00	1.68E-10
18	2.89263E-18	6.66E-16	0.00	1.5E-11
19	1.27806E-19	3.11E-17	0.00	1.26E-12
20	5.36456E-21	1.37E-18	0.00	1E-13

Ducks	Risk Sum	11.75228		
15	Max Risk	28.43498		
Gamma Alpha "α" =	5			
Gamma Beta "β" =	151			
Gamma Altitude Band %	0.323775914			
Total of specific species in AO	440			
λ'	4.67087E-07			
Risk Multiplier	37			
			MAX	
n	P(n)	Risk	P(n)	Risk
0	0.470712534	0	0.22	0
1	0.35468552	6.946032	0.33	9.61
2	0.133629136	3.377504	0.25	9.701325
3	0.033563528	1.102446	0.13	5.489926
4	0.006322594	0.267607	0.05	2.417147
5	0.000952825	0.050092	0.01	0.870483
6	0.00011966	0.007542	0.00	0.259628
7	1.28807E-05	0.000947	0.00	0.065243
8	1.21321E-06	0.000102	0.00	0.014094
9	1.01574E-07	9.6E-06	0.00	0.002666
10	7.65367E-09	8.04E-07	0.00	0.000448
11	5.24282E-10	6.06E-08	0.00	6.78E-05
12	3.29209E-11	4.15E-09	0.00	9.33E-06
13	1.90816E-12	2.61E-10	0.00	1.18E-06
14	1.02701E-13	1.51E-11	0.00	1.37E-07
15	5.15907E-15	8.13E-13	0.00	1.48E-08
16	2.42963E-16	4.08E-14	0.00	1.49E-09
17	1.07691E-17	1.92E-15	0.00	1.41E-10
18	4.5081E-19	8.52E-17	0.00	1.26E-11
19	1.78783E-20	3.57E-18	0.00	1.06E-12
20	6.73574E-22	1.41E-19	0.00	8.42E-14

Eagles	Risk Sum	22.47843		
16	Max Risk	23.8239		
Gamma Alpha "α" =	4.8			
Gamma Beta "β" =	153			
Gamma Altitude Band %	0.340323459			
Total of specific species in AO	800			
λ'	8.92652E-07			
Risk Multiplier	31			
			MAX	
n	P(n)	Risk	P(n)	Risk
0	0.23692001	0	0.22	0
1	0.341172563	8.070601	0.33	8.05
2	0.245649824	7.656846	0.25	8.128137
3	0.117914594	4.158773	0.13	4.599668
4	0.042450218	1.760869	0.05	2.025178
5	0.01222594	0.607017	0.01	0.729323
6	0.002934292	0.172687	0.00	0.217526
7	0.00060364	0.041324	0.00	0.054663
8	0.000108658	0.008496	0.00	0.011808
9	1.73856E-05	0.001529	0.00	0.002233
10	2.50358E-06	0.000245	0.00	0.000376
11	3.27749E-07	3.52E-05	0.00	5.68E-05
12	3.93308E-08	4.61E-06	0.00	7.82E-06
13	4.35674E-09	5.53E-07	0.00	9.86E-07
14	4.48132E-10	6.13E-08	0.00	1.15E-07
15	4.30217E-11	6.31E-09	0.00	1.24E-08
16	3.87204E-12	6.05E-10	0.00	1.25E-09
17	3.27992E-13	5.45E-11	0.00	1.18E-10
18	2.624E-14	4.62E-12	0.00	1.05E-11
19	1.98876E-15	3.69E-13	0.00	8.86E-13
20	1.43194E-16	2.8E-14	0.00	7.06E-14

Hawks	Risk Sum	2.715544		
17	Max Risk	19.21282		
Gamma Alpha "α" =	4.6			
Gamma Beta "β" =	155			
Gamma Altitude Band %	0.356522721			
Total of specific species in AO	200			
λ'	2.33785E-07			
Risk Multiplier	25			
			MAX	
n	P(n)	Risk	P(n)	Risk
0	0.685817019	0	0.22	0
1	0.258652064	2.031602	0.33	6.50
2	0.048774592	0.568026	0.25	6.554949
3	0.006131688	0.10189	0.13	3.709409
4	0.000578133	0.01273	0.05	1.633208
5	4.36079E-05	0.0012	0.01	0.588164
6	2.74108E-06	9.05E-05	0.00	0.175425
7	1.47683E-07	5.69E-06	0.00	0.044083
8	6.96224E-09	3.06E-07	0.00	0.009523
9	2.91752E-10	1.44E-08	0.00	0.001801
10	1.10033E-11	6.05E-10	0.00	0.000303
11	3.77257E-13	2.28E-11	0.00	4.58E-05
12	1.18567E-14	7.83E-13	0.00	6.31E-06
13	3.43976E-16	2.46E-14	0.00	7.95E-07
14	9.26633E-18	7.14E-16	0.00	9.26E-08
15	2.32983E-19	1.92E-17	0.00	1E-08
16	5.49176E-21	4.83E-19	0.00	1.01E-09
17	1.21835E-22	1.14E-20	0.00	9.55E-11
18	2.55273E-24	2.53E-22	0.00	8.5E-12
19	5.0671E-26	5.3E-24	0.00	7.15E-13
20	9.55515E-28	1.05E-25	0.00	5.69E-14

Rock Dove	Risk Sum	2.426949		
18	Max Risk	18.44431		
Gamma Alpha "α" =	4.4			
Gamma Beta "β" =	141			
Gamma Altitude Band %	0.427237501			
Total of specific species in AO	160			
λ'	2.24125E-07			
Risk Multiplier	24			
			MAX	
n	P(n)	Risk	P(n)	Risk
0	0.696589117	0	0.22	0
1	0.251858444	1.833998	0.33	6.24
2	0.045530912	0.496092	0.25	6.292751
3	0.005487379	0.0856	0.13	3.561033
4	0.000496004	0.01026	0.05	1.567879
5	3.5867E-05	0.000927	0.01	0.564638
6	2.16134E-06	6.7E-05	0.00	0.168408
7	1.11636E-07	4.04E-06	0.00	0.04232
8	5.04539E-09	2.09E-07	0.00	0.009142
9	2.0269E-10	9.43E-09	0.00	0.001729
10	7.32845E-12	3.79E-10	0.00	0.000291
11	2.40879E-13	1.37E-11	0.00	4.4E-05
12	7.25768E-15	4.5E-13	0.00	6.05E-06
13	2.01853E-16	1.36E-14	0.00	7.63E-07
14	5.21298E-18	3.77E-16	0.00	8.89E-08
15	1.25653E-19	9.74E-18	0.00	9.61E-09
16	2.83945E-21	2.35E-19	0.00	9.69E-10
17	6.039E-23	5.31E-21	0.00	9.17E-11
18	1.21303E-24	1.13E-22	0.00	8.16E-12
19	2.30834E-26	2.27E-24	0.00	6.86E-13
20	4.173E-28	4.31E-26	0.00	5.46E-14

Gulls	Risk Sum	2.292915		
19	Max Risk	16.90728		
Gamma Alpha "α" =	4.2			
Gamma Beta "β" =	159			
Gamma Altitude Band %	0.386576789			
Total of specific species in AO	180			
λ'	2.28144E-07			
Risk Multiplier	22			
			MAX	
n	P(n)	Risk	P(n)	Risk
0	0.692087305	0	0.22	0
1	0.254718004	1.72548	0.33	5.72
2	0.046873611	0.473292	0.25	5.768355
3	0.005750504	0.083013	0.13	3.26428
4	0.000529108	0.010126	0.05	1.437223
5	3.89469E-05	0.000931	0.01	0.517584
6	2.38903E-06	6.85E-05	0.00	0.154374
7	1.25609E-07	4.2E-06	0.00	0.038793
8	5.7787E-09	2.21E-07	0.00	0.00838
9	2.36313E-10	1.02E-08	0.00	0.001585
10	8.69732E-12	4.16E-10	0.00	0.000267
11	2.90999E-13	1.53E-11	0.00	4.03E-05
12	8.92502E-15	5.12E-13	0.00	5.55E-06
13	2.52676E-16	1.57E-14	0.00	7E-07
14	6.64256E-18	4.45E-16	0.00	8.15E-08
15	1.62983E-19	1.17E-17	0.00	8.8E-09
16	3.74905E-21	2.87E-19	0.00	8.88E-10
17	8.11655E-23	6.6E-21	0.00	8.4E-11
18	1.65958E-24	1.43E-22	0.00	7.48E-12
19	3.21472E-26	2.92E-24	0.00	6.29E-13
20	5.91577E-28	5.66E-26	0.00	5.01E-14

Herons	Risk Sum	5.028598		
20	Max Risk	16.90728		
Gamma Alpha "α" =	4			
Gamma Beta "β" =	141			
Gamma Altitude Band %	0.454207518			
Total of specific species in AO	250			
λ'	3.72301E-07			
Risk Multiplier	22			
			MAX	
n	P(n)	Risk	P(n)	Risk
0	0.548482842	0	0.22	0
1	0.329418401	3.272237	0.33	5.72
2	0.098924227	1.317896	0.25	5.768355
3	0.019804607	0.356613	0.13	3.26428
4	0.002973658	0.06998	0.05	1.437223
5	0.000357195	0.010476	0.01	0.517584
6	3.57552E-05	0.001258	0.00	0.154374
7	3.06779E-06	0.000126	0.00	0.038793
8	2.30314E-07	1.08E-05	0.00	0.00838
9	1.53696E-08	8.11E-07	0.00	0.001585
10	9.23099E-10	5.41E-08	0.00	0.000267
11	5.04011E-11	3.25E-09	0.00	4.03E-05
12	2.52257E-12	1.77E-10	0.00	5.55E-06
13	1.16543E-13	8.88E-12	0.00	7E-07
14	4.99968E-15	4.1E-13	0.00	8.15E-08
15	2.00187E-16	1.76E-14	0.00	8.8E-09
16	7.51451E-18	7.05E-16	0.00	8.88E-10
17	2.65483E-19	2.65E-17	0.00	8.4E-11
18	8.85826E-21	9.35E-19	0.00	7.48E-12
19	2.80014E-22	3.12E-20	0.00	6.29E-13
20	8.40881E-24	9.86E-22	0.00	5.01E-14

Mourning Dove	Risk Sum	6.170355		
21	Max Risk	13.06472		
Gamma Alpha "α" =	3.8			
Gamma Beta "β" =	141			
Gamma Altitude Band %	0.461588324			
Total of specific species in AO	340			
λ'	5.14557E-07			
Risk Multiplier	17			
			MAX	
n	P(n)	Risk	P(n)	Risk
0	0.436010851	0	0.22	0
1	0.36192744	3.470094	0.33	4.42
2	0.150215839	1.837957	0.25	4.457366
3	0.041564129	0.648244	0.13	2.522398
4	0.008625473	0.171911	0.05	1.110581
5	0.001431981	0.035368	0.01	0.399952
6	0.000198112	0.005863	0.00	0.119289
7	2.34929E-05	0.000811	0.00	0.029977
8	2.43765E-06	9.62E-05	0.00	0.006475
9	2.24829E-07	9.98E-06	0.00	0.001225
10	1.86628E-08	9.2E-07	0.00	0.000206
11	1.40834E-09	7.64E-08	0.00	3.12E-05
12	9.74206E-11	5.77E-09	0.00	4.29E-06
13	6.22059E-12	3.99E-10	0.00	5.41E-07
14	3.68831E-13	2.55E-11	0.00	6.29E-08
15	2.04108E-14	1.51E-12	0.00	6.8E-09
16	1.05892E-15	8.36E-14	0.00	6.86E-10
17	5.17059E-17	4.33E-15	0.00	6.49E-11
18	2.38447E-18	2.12E-16	0.00	5.78E-12
19	1.04175E-19	9.76E-18	0.00	4.86E-13
20	4.32371E-21	4.26E-19	0.00	3.87E-14

Owls	Risk Sum	8.654626		
22	Max Risk	12.29621		
Gamma Alpha "α" =	3.6			
Gamma Beta "β" =	141			
Gamma Altitude Band %	0.46417771			
Total of specific species in AO	460			
λ'	7.00071E-07			
Risk Multiplier	16			
			MAX	
n	P(n)	Risk	P(n)	Risk
0	0.323239902	0	0.22	0
1	0.365054378	3.952868	0.33	4.16
2	0.206138997	2.834532	0.25	4.195168
3	0.077601747	1.270657	0.13	2.374022
4	0.021910087	0.441223	0.05	1.045253
5	0.004948877	0.121846	0.01	0.376425
6	0.000931511	0.027385	0.00	0.112272
7	0.000150287	0.00515	0.00	0.028213
8	2.12161E-05	0.000831	0.00	0.006095
9	2.66229E-06	0.000117	0.00	0.001153
10	3.00668E-07	1.47E-05	0.00	0.000194
11	3.08694E-08	1.66E-06	0.00	2.93E-05
12	2.90522E-09	1.71E-07	0.00	4.04E-06
13	2.52388E-10	1.61E-08	0.00	5.09E-07
14	2.03598E-11	1.4E-09	0.00	5.92E-08
15	1.5329E-12	1.13E-10	0.00	6.4E-09
16	1.082E-13	8.47E-12	0.00	6.46E-10
17	7.18804E-15	5.98E-13	0.00	6.11E-11
18	4.50994E-16	3.97E-14	0.00	5.44E-12
19	2.68071E-17	2.49E-15	0.00	4.57E-13
20	1.51374E-18	1.48E-16	0.00	3.64E-14

American Kestrel	Risk Sum	8.533788		
23	Max Risk	10.75918		
Gamma Alpha "α" =	3.4			
Gamma Beta "β" =	141			
Gamma Altitude Band %	0.461544896			
Total of specific species in AO	510			
λ'	7.71764E-07			
Risk Multiplier	14			
			MAX	
n	P(n)	Risk	P(n)	Risk
0	0.287936567	0	0.22	0
1	0.358485367	3.5737	0.33	3.64
2	0.223159843	2.854298	0.25	3.670772
3	0.092612456	1.380307	0.13	2.077269
4	0.028825976	0.519783	0.05	0.914596
5	0.007177755	0.157121	0.01	0.329372
6	0.001489402	0.038843	0.00	0.098238
7	0.000264904	0.008048	0.00	0.024687
8	4.12262E-05	0.001431	0.00	0.005333
9	5.70303E-06	0.000223	0.00	0.001009
10	7.10035E-07	3.08E-05	0.00	0.00017
11	8.03641E-08	3.84E-06	0.00	2.57E-05
12	8.33787E-09	4.34E-07	0.00	3.53E-06
13	7.98521E-10	4.5E-08	0.00	4.45E-07
14	7.10122E-11	4.31E-09	0.00	5.18E-08
15	5.89408E-12	3.84E-10	0.00	5.6E-09
16	4.58639E-13	3.18E-11	0.00	5.65E-10
17	3.3589E-14	2.48E-12	0.00	5.35E-11
18	2.32327E-15	1.81E-13	0.00	4.76E-12
19	1.52237E-16	1.25E-14	0.00	4E-13
20	9.47686E-18	8.22E-16	0.00	3.19E-14

Shorebirds	Risk Sum	9.715235		
24	Max Risk	9.222155		
Gamma Alpha "α" =	3.2			
Gamma Beta "β" =	141			
Gamma Altitude Band %	0.453387107			
Total of specific species in AO	660			
λ'	9.811E-07			
Risk Multiplier	12			
			MAX	
n	P(n)	Risk	P(n)	Risk
0	0.205415987	0	0.22	0
1	0.325115611	3.09998	0.33	3.12
2	0.257283189	3.311249	0.25	3.146376
3	0.135735591	1.946206	0.13	1.780517
4	0.053707796	0.887396	0.05	0.78394
5	0.017000861	0.332266	0.01	0.282319
6	0.004484595	0.103389	0.00	0.084204
7	0.001013979	0.02715	0.00	0.02116
8	0.000200605	0.006133	0.00	0.004571
9	3.5278E-05	0.001213	0.00	0.000865
10	5.58351E-06	0.000213	0.00	0.000145
11	8.03374E-07	3.38E-05	0.00	2.2E-05
12	1.0596E-07	4.86E-06	0.00	3.03E-06
13	1.29003E-08	6.41E-07	0.00	3.82E-07
14	1.4584E-09	7.8E-08	0.00	4.44E-08
15	1.53882E-10	8.82E-09	0.00	4.8E-09
16	1.5222E-11	9.3E-10	0.00	4.85E-10
17	1.41719E-12	9.2E-11	0.00	4.58E-11
18	1.24611E-13	8.57E-12	0.00	4.08E-12
19	1.03802E-14	7.53E-13	0.00	3.43E-13
20	8.2145E-16	6.28E-14	0.00	2.73E-14

Crows - Ravens	Risk Sum	11.35568		
25	Max Risk	9.222155		
Gamma Alpha "α" =	3			
Gamma Beta "β" =	141			
Gamma Altitude Band %	0.439561583			
Total of specific species in AO	780			
λ'	1.12412E-06			
Risk Multiplier	12			
			MAX	
n	P(n)	Risk	P(n)	Risk
0	0.16309103	0	0.22	0
1	0.295756901	2.970259	0.33	3.12
2	0.268169698	3.79334	0.25	3.146376
3	0.162103824	2.517138	0.13	1.780517
4	0.073491664	1.274914	0.05	0.78394
5	0.026654644	0.535519	0.01	0.282319
6	0.00805613	0.18905	0.00	0.084204
7	0.002087052	0.056678	0.00	0.02116
8	0.000473095	0.014653	0.00	0.004571
9	9.53258E-05	0.00332	0.00	0.000865
10	1.72868E-05	0.000669	0.00	0.000145
11	2.84988E-06	0.000121	0.00	2.2E-05
12	4.30676E-07	2E-05	0.00	3.03E-06
13	6.00776E-08	3.02E-06	0.00	3.82E-07
14	7.78196E-09	4.22E-07	0.00	4.44E-08
15	9.40812E-10	5.46E-08	0.00	4.8E-09
16	1.06632E-10	6.6E-09	0.00	4.85E-10
17	1.13748E-11	7.48E-10	0.00	4.58E-11
18	1.14598E-12	7.98E-11	0.00	4.08E-12
19	1.09377E-13	8.04E-12	0.00	3.43E-13
20	9.91749E-15	7.67E-13	0.00	2.73E-14

Blackbirds - Starling	Risk Sum	9.274724		
26	Max Risk	6.916616		
Gamma Alpha "α" =	2.8			
Gamma Beta "β" =	141			
Gamma Altitude Band %	0.420114193			
Total of specific species in AO	880			
λ'	1.21213E-06			
Risk Multiplier	9			
			MAX	
n	P(n)	Risk	P(n)	Risk
0	0.141504743	0	0.22	0
1	0.276701493	2.137922	0.33	2.34
2	0.2705341	3.023779	0.25	2.359782
3	0.176336114	2.15658	0.13	1.335387
4	0.086202881	1.157803	0.05	0.587955
5	0.033712603	0.517208	0.01	0.211739
6	0.010987061	0.195453	0.00	0.063153
7	0.003069192	0.062999	0.00	0.01587
8	0.000750196	0.017544	0.00	0.003428
9	0.000162994	0.004285	0.00	0.000648
10	3.18723E-05	0.000931	0.00	0.000109
11	5.66579E-06	0.000182	0.00	1.65E-05
12	9.23252E-07	3.24E-05	0.00	2.27E-06
13	1.38873E-07	5.27E-06	0.00	2.86E-07
14	1.93968E-08	7.93E-07	0.00	3.33E-08
15	2.52859E-09	1.11E-07	0.00	3.6E-09
16	3.09029E-10	1.44E-08	0.00	3.63E-10
17	3.5546E-11	1.76E-09	0.00	3.44E-11
18	3.86153E-12	2.03E-10	0.00	3.06E-12
19	3.97417E-13	2.21E-11	0.00	2.57E-13
20	3.88559E-14	2.27E-12	0.00	2.05E-14

Sparrows	Risk Sum	0.392892		
27	Max Risk	3.074052		
Gamma Alpha "α" =	2.6			
Gamma Beta "β" =	141			
Gamma Altitude Band %	0.395301966			
Total of specific species in AO	170			
λ'	2.20332E-07			
Risk Multiplier	4			
			MAX	
n	P(n)	Risk	P(n)	Risk
0	0.700863803	0	0.22	0
1	0.249116222	0.298079	0.33	1.04
2	0.044273147	0.079556	0.25	1.048792
3	0.005245507	0.013513	0.13	0.593506
4	0.000466118	0.001593	0.05	0.261313
5	3.31356E-05	0.000141	0.01	0.094106
6	1.96296E-06	1.01E-05	0.00	0.028068
7	9.9674E-08	5.96E-07	0.00	0.007053
8	4.42854E-09	3.02E-08	0.00	0.001524
9	1.74898E-10	1.34E-09	0.00	0.000288
10	6.21662E-12	5.31E-11	0.00	4.85E-05
11	2.00877E-13	1.89E-12	0.00	7.33E-06
12	5.95E-15	6.1E-14	0.00	1.01E-06
13	1.62683E-16	1.81E-15	0.00	1.27E-07
14	4.13031E-18	4.94E-17	0.00	1.48E-08
15	9.78723E-20	1.25E-18	0.00	1.6E-09
16	2.17424E-21	2.97E-20	0.00	1.62E-10
17	4.54598E-23	6.6E-22	0.00	1.53E-11
18	8.97684E-25	1.38E-23	0.00	1.36E-12
19	1.67934E-26	2.72E-25	0.00	1.14E-13
20	2.98453E-28	5.1E-27	0.00	9.11E-15

Swallows	Risk Sum	1.760176		
28	Max Risk	1.537026		
Gamma Alpha "α" =	2.4			
Gamma Beta "β" =	141			
Gamma Altitude Band %	0.365606619			
Total of specific species in AO	880			
λ'	1.05486E-06			
Risk Multiplier	2			
			MAX	
n	P(n)	Risk	P(n)	Risk
0	0.182370222	0	0.22	0
1	0.310342411	0.50749	0.33	0.52
2	0.264057396	0.595592	0.25	0.524396
3	0.149783607	0.372949	0.13	0.296753
4	0.063722308	0.179864	0.05	0.130657
5	0.02168746	0.071644	0.01	0.047053
6	0.006150985	0.023855	0.00	0.014034
7	0.001495319	0.006724	0.00	0.003527
8	0.000318076	0.001632	0.00	0.000762
9	6.01417E-05	0.000347	0.00	0.000144
10	1.02344E-05	6.56E-05	0.00	2.42E-05
11	1.58328E-06	1.12E-05	0.00	3.67E-06
12	2.24524E-07	1.73E-06	0.00	5.04E-07
13	2.93905E-08	2.45E-07	0.00	6.36E-08
14	3.57245E-09	3.21E-08	0.00	7.4E-09
15	4.05287E-10	3.9E-09	0.00	8E-10
16	4.31052E-11	4.42E-10	0.00	8.08E-11
17	4.31487E-12	4.7E-11	0.00	7.64E-12
18	4.07927E-13	4.71E-12	0.00	6.8E-13
19	3.65356E-14	4.45E-13	0.00	5.72E-14
20	3.10866E-15	3.99E-14	0.00	4.55E-15

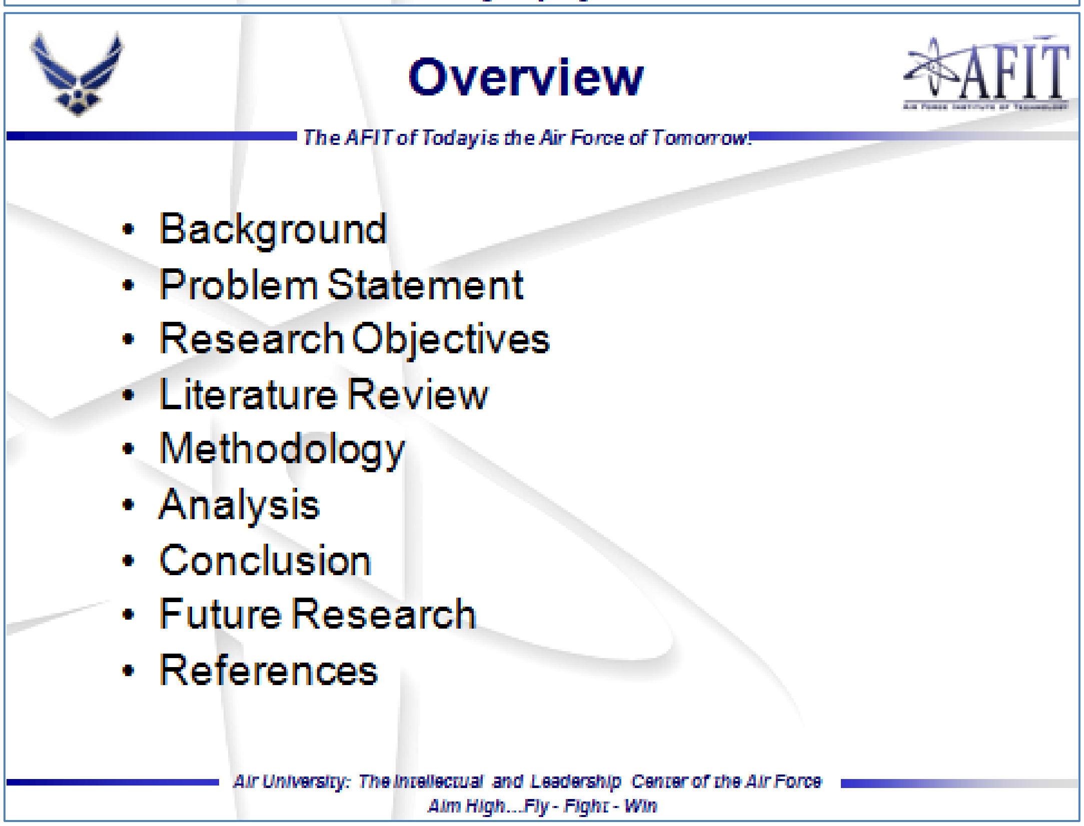
8. Appendix II - Numerical Matrix Exponentiation Code

The following is VBA code for numerical calculation of the Extended Spatial Poisson Process:

```
Option Explicit
Public Sub EXPO()
Dim iRow As Integer, iColumn As Integer, iIteration As Integer
Dim i As Long, N As Long
Dim V As Long
Dim Lambda As Double
Dim vDistance As Double
Dim vLength As Double
Dim vHeight As Double
Dim aircraftA As Double
Dim arrayA() As Variant 'A matrix
Dim arrayI() As Variant 'I Matrix
Dim arrayQ() As Variant
Dim arrayPower() As Variant
Dim arrayPowerFact() As Variant

Application.ScreenUpdating = False

'Input
N = Sheet1.Range("B1") '# of birds in u(S)
V = Sheet1.Range("B2") 'volume of AO
iIteration = Sheet1.Range("B3") 'Iteration for series
vDistance = Sheet1.Range("B4") 'distance aircraft travels in AO
vLength = Sheet1.Range("B5") 'wingspan
vHeight = Sheet1.Range("B6") 'height

'Initialize Variables
Lambda = -1 * Log(1 - (1 / V))
aircraftA = 0.5 * vLength * 0.5 * vHeight * vDistance * 3.14

Sheet2.Select
Clear_Sheet

'Build Identity Matrix
ReDim arrayI(1 To N + 1, 1 To N + 1)
For iRow = 1 To N + 1
    For iColumn = 1 To N + 1

        If iRow = iColumn Then
            arrayI(iRow, iColumn) = 1
        Else
            arrayI(iRow, iColumn) = 0
        End If
    Next
Next


'Build A Matrix

ReDim arrayA(1 To N + 1, 1 To N + 1)
For iRow = 1 To N + 1
    For iColumn = 1 To N + 1
        If iRow = iColumn Then
            arrayA(iRow, iColumn) = -1 * (N - (iRow - 1)) * Lambda * aircraftA
        ElseIf iColumn = iRow + 1 Then
            arrayA(iRow, iColumn) = (N - (iRow - 1)) * Lambda * aircraftA
        ElseIf iColumn = iRow + 1 Then
        Else
            arrayA(iRow, iColumn) = 0
        End If
    Next
Next


'Initial summation I + A
ReDim arrayQ(1 To N + 1, 1 To N + 1)
For iRow = 1 To N + 1
    For iColumn = 1 To N + 1
        arrayQ(iRow, iColumn) = arrayI(iRow, iColumn) + arrayA(iRow, iColumn)
    Next
Next
```

```
'Initial step matrix calculation -- A squared

ReDim arrayPower(1 To N + 1, 1 To N + 1)

arrayPower() = Application.WorksheetFunction.MMult(arrayA, arrayA)


'divide by 2!
ReDim arrayPowerFact(1 To N + 1, 1 To N + 1)
For iRow = 1 To N + 1
    For iColumn = 1 To N + 1
        arrayPowerFact(iRow, iColumn) = arrayPower(iRow, iColumn) / Application.WorksheetFunction.Fact
(2)
    Next
Next

'Add I + A + (A^2)/2!
For iRow = 1 To N + 1
    For iColumn = 1 To N + 1
        arrayQ(iRow, iColumn) = arrayQ(iRow, iColumn) + arrayPowerFact(iRow, iColumn)
    Next
Next

'Subsequent step matrix calculations
For i = 3 To iIteration

    'take next power of A
    arrayPower() = Application.WorksheetFunction.MMult(arrayPower, arrayA)

    'divide by i!
    For iRow = 1 To N + 1
        For iColumn = 1 To N + 1
            arrayPowerFact(iRow, iColumn) = arrayPower(iRow, iColumn) / Application.WorksheetFunction.
Fact(i)
        Next
    Next

    'add to Q
    For iRow = 1 To N + 1
        For iColumn = 1 To N + 1
            arrayQ(iRow, iColumn) = arrayQ(iRow, iColumn) + arrayPowerFact(iRow, iColumn)
        Next
    Next

Next

For iRow = 1 To N + 1
    For iColumn = 1 To N + 1
            Sheet2.Cells(iRow, iColumn) = arrayQ(iRow, iColumn)
    Next
Next

Application.ScreenUpdating = True
Sheet2.Select

End Sub
```

9. Appendix III - Road Show

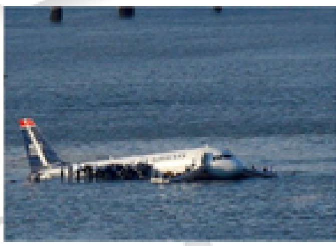


Background

The AFIT of Today is the Air Force of Tomorrow!

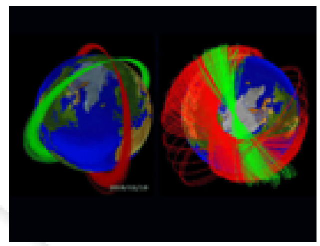
Random collisions of relatively small numbers of entities in very large, yet bounded, spaces are rare, but not impossible!

15 Jan 2009 | 3 Feb 2009 | 10 Feb 2009



US Airways flight 1549

HMS Vanguard & Le Triomphant



Iridium 33 and Kosmos 2251

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Background

The AFIT of Today is the Air Force of Tomorrow!

- ❑ Federal Aviation Administration (FAA) data from1990 - 2010
  - ❑ 121,000 (civil and U.S. Air Force) wildlife strikes
- ❑ United States Air Force (USAF) data from 1985 - 2011
  - ❑ 95,383 wildlife strikes
  - ❑ 33 fatalities
  - ❑ 39 aircraft lost
  - ❑ Damages > $820M
- ❑ Stakeholders
  - ❑ FAA
  - ❑ DoD
  - ❑ NASA
  - ❑ DOE (specifically wind energy)
  - ❑ Dept of Fish and Game/Wildlife

** U.S. Air Force Safety Center (2012)*

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- Federal Aviation Administration (FAA)
  - 92% of the bird strikes to commercial aircraft occur at or below 3,500 ft above ground level (AGL)

- United States Air Force (USAF)
  - 96.72% of its bird strikes below 3,500 ft AGL.

USAF Wildlife Strikes by Phase of Operation (1995 – 2011)

Phase of Flight	Cost	% of Total	Count	% of Total
Takeoff/Initial Climb	$137,035,238	32.16%	7299	12.25%
Enroute/Air Work/Air-to-Air/Air Refueling	$19,183,980	4.50%	2830	4.75%
Flight Demonstration	$1,878,791	0.44%	27	0.05%
Low Level/Air-to-Ground/Air Delivery	$174,151,930	40.87%	6949	11.66%
Hover	$0	0.00%	10	0.02%
Traffic Pattern/Go-Around	$31,446,708	7.38%	7618	12.78%
Initial Approach/Final Approach/Landing	$46,825,552	10.99%	16048	26.92%
Parked/Ground Ops	$3,561,521	0.84%	423	0.71%
Unknown	$12,079,198	2.83%	18400	30.87%

** U.S. Air Force Safety Center (2012)*

- Current Advisory Systems

Avian Hazard Advisory System (AHAS)

AHAS RISK FOR VR042

VR042					
SEGMENT	DateTime	NEXRAD	AHAS RISK	BASED ON	HEIGHT (100ft AGL)
VR042A-B	2012/05/18 2:33Z	LOW	LOW	NEXRAD	NA
VR042B-C	2012/05/18 2:33Z	LOW	LOW	NEXRAD	NA
VR042C-D	2012/05/18 2:33Z	LOW	LOW	NEXRAD	NA
VR042D-E	2012/05/18 2:33Z	LOW	LOW	NEXRAD	NA
VR042E-F	2012/05/18 2:33Z	LOW	LOW	NEXRAD	NA
VR042F-G	2012/05/18 2:33Z	MODERATE	MODERATE	NEXRAD	NA
VR042G-H	2012/05/18 2:33Z	LOW	LOW	NEXRAD	NA
VR042H-I	2012/05/18 2:33Z	SEVERE	SEVERE	NEXRAD	NA
VR042I-J	2012/05/18 2:33Z	SEVERE	SEVERE	NEXRAD	NA
VR042J-K	2012/05/18 2:33Z	MODERATE	MODERATE	NEXRAD	NA
VR042K-L	2012/05/18 2:33Z	MODERATE	MODERATE	NEXRAD	NA
VR042L-M	2012/05/18 2:33Z	LOW	LOW	NEXRAD	NA
VR042M-N	2012/05/18 2:33Z	LOW	LOW	NEXRAD	NA
VR042N-O	2012/05/18 2:33Z	LOW	LOW	NEXRAD	NA

Alternate Routes: VR071, VR096, VR1061, VR1754, VR073

HAZARDS

DAMS:

VR042E-F: BEAVER CREEK DAM

VR042E-F: CHERRYSTONE CREEK DAM #1

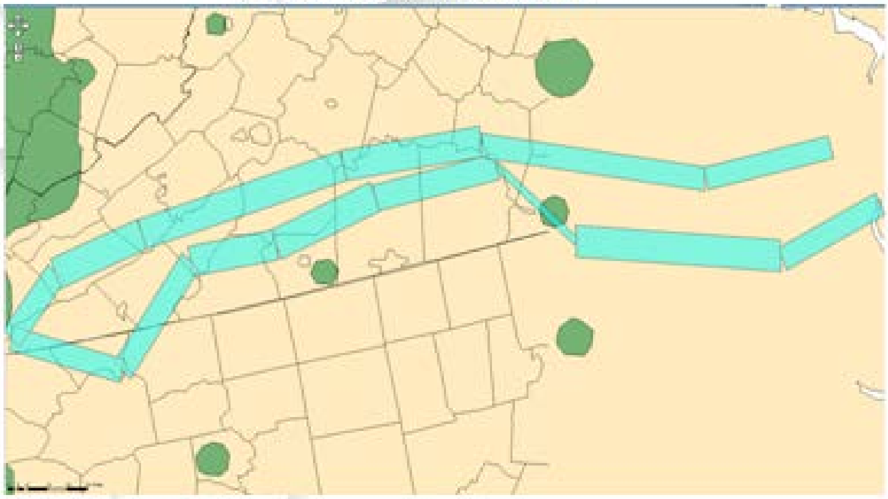
** United States Air Force BASH (2012)*

Background

The AFIT of Today is the Air Force of Tomorrow!

- ❑ Current Advisory Systems

Bird Avoidance Model (BAM)



* United States Air Force BASH (2012)

Problem Statement

The AFIT of Today is the Air Force of Tomorrow!

- The purpose of this research is to create a closed form mathematical model to determine the probability of an aircraft/wildlife collision encounter (strike) in a defined space (e.g. on a segment of a Low Level route/Air-to-Ground/Air Delivery) with a set of given parameters.
- Generate a risk score associated with the encounter(s).
- Develop a Decision Support System (DSS) in order to underpin decisions by pilots and planners when coupled with existing alert and advisory systems.

Research Objectives

The AFIT of Today is the Air Force of Tomorrow!

- Study the systems that provide avian detection, alerts and advisories; evaluate the level of fidelity they provide.
- Determine avian characteristics, geographical and environmental conditions, and other factors that drive avian population densities and spatial patterns.
- Conduct research on Spatial Poisson Processes and relate them to aircraft/avian encounters for a defined space.
- Highlight and compare existing aircraft/avian collision mathematical models.
- Conduct research on Extended Spatial Poisson Processes and how they relate to encounter models.
- Implement techniques for performing matrix exponentiation.

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Literature Review

The AFIT of Today is the Air Force of Tomorrow!

- Avian Characteristics and Environmental factors that influence populations
  - Land use practices
  - Water management facilities
  - Agricultural activities
- Avian movements can be partitioned into three categories: migrating birds, commuting birds and resident birds.
- During peak migration in May and September densities can be as high as 1500 or more birds per cubic kilometer.

* FAA (2010)

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Literature Review

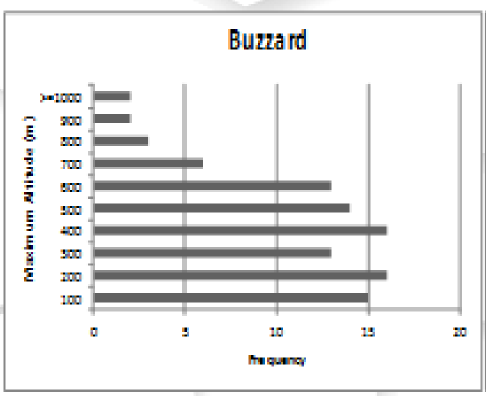
The AFIT of Today is the Air Force of Tomorrow!

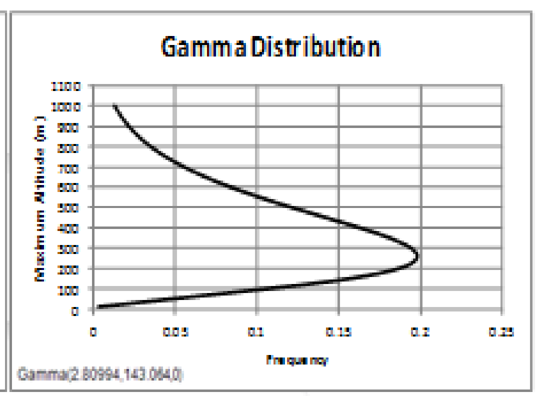
- Land use practices
  - Waste disposal operations
  - Trash Transfer Stations
  - Recycling centers
  - Construction
  - Golf Courses
- Water management facilities
  - Storm water management facilities
  - Wastewater treatment facilities
  - Artificial marshes
  - Wastewater discharge and sludge disposal
- Agricultural activities
  - Crop production
  - Livestock production
  - Aquaculture

Literature Review

The AFIT of Today is the Air Force of Tomorrow!

- Avian Altitude Density Distributions
  - Various literatures all reported that the gamma distribution is good fit for bird height distribution.





* *Shamoun-Baranes, et al. (2005) & Stumpf, et al. (2011)*

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Literature Review

The AFIT of Today is the Air Force of Tomorrow!

- Avian Detection and Advisory
  - Radar has been used extensively to monitor bird movements and warn the relevant personnel
  - Radar used to monitor bird movements in the U.S. is the WSR-88D NEXRAD radar.

* Sodhi (2002), Dokter, et al. (2011)

Literature Review

The AFIT of Today is the Air Force of Tomorrow!

- Risk Level Ranking
  - Criteria
    - Damage
    - Major Damage
    - Effect on Flight
  - All pairwise ranked
  - Composite score
  - Strongly correlated to mass of bird

	Ranking by Criteria				
Species Group	Damage	Major Damage	Effect on flight	Composite Ranking	Relative Hazard Score
Deer	1	1	1	1	100
Vultures	2	2	2	2	65
Geese	3	3	4	3	52
Cranes	4	4	7	4	48
Osprey	6	5	5	5	50
Pelicans	5	7	5	6	44
Ducks	7	6	8	7	37
Hawks	8	15	9	9	25
Eagles	8	15	9	9	31
Rock Dove	11	8	11	10	24
Gulls	10	11	13	11	22
Herons	12	14	12	12	22
Mourning Dove	14	9	17	13	17
Owls	15	12	19	14	16
Coyote	15	17	6	15	20
American Kestrel	16	10	16	16	14
Shorebirds	17	19	14	17	12
Crows - Ravens	18	18	15	18	12
Blackbirds - Starling	19	18	18	19	9
Sparrows	20	21	20	20	4
Swallows	21	20	21	21	2

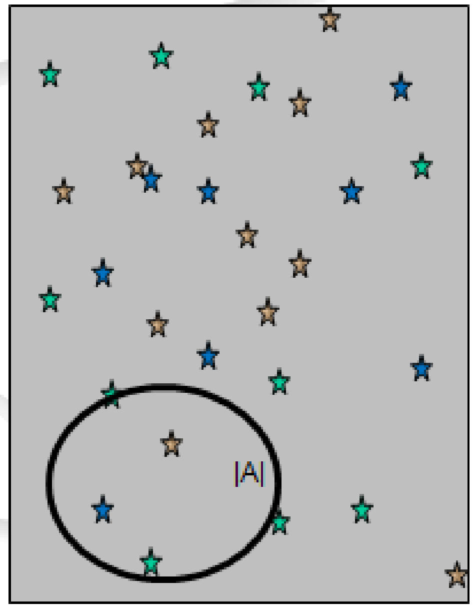
* Dolbeer, Wright and Cleary (2000)

Literature Review

The AFIT of Today is the Air Force of Tomorrow!

- Spatial Poisson Process
  - One of the simplest and fundamental spatial point processes is the completely random, or Spatial Poisson Point Process.
  - It possesses the property of "no interaction" between points or "complete spatial randomness"

$$p_n = e^{-\lambda * |A|} * \frac{(\lambda * |A|)^n}{n!}$$



* *Møller and Waagepeterson (2006)*

Literature Review

The AFIT of Today is the Air Force of Tomorrow!

- Extended Spatial Poisson Process
  - Based upon generalizing the simplest Markov birth process, the Poisson process.
  - ESPP's involve representing a discrete distribution as the distribution of the number of events occurring in a finite time interval of a state-dependent Markov birth-death process.
  - Holzmann and Cochran (2012) furthered Faddy's studies of spatial data with respect to Extended SPP.
  - They utilize a linearly decreasing arrival rate for the birth-death process of the Markov transition in order to remove encounter-able elements from the space.

* *Podlich, Faddy and Smyth (1999) & Holzmann and Cochran (2012)*

Literature Review

- Extended Spatial Poisson Process

$$Q = \begin{bmatrix} -N\lambda & N\lambda & 0 & \dots & 0 \\ 0 & -(N-1)\lambda & (N-1)\lambda & \ddots & 0 \\ 0 & 0 & -(N-2)\lambda & \ddots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Where $\lambda = \ln\left(1 - \frac{1}{|V|}\right)$ and N = # of entities in bounded region

This then leads to the probability distribution for the number of encounters in the AO for the specific window |A| as:

$$p = p_0 \exp(Q * |A|)$$

* Holzmann and Cochran (2012)

Literature Review

- Extended Spatial Poisson Process Example

$$P_{ij} = \begin{bmatrix} 0.095 & 0.284 & 0.346 & 0.205 & 0.063 & 0.007 \\ 0 & 0.152 & 0.366 & 0.33 & 0.132 & 0.02 \\ 0 & 0 & 0.243 & 0.439 & 0.264 & 0.053 \\ 0 & 0 & 0 & 0.39 & 0.469 & 0.141 \\ 0 & 0 & 0 & 0 & 0.624 & 0.376 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$p = \begin{bmatrix} 0.095 & 0.284 & 0.346 & 0.205 & 0.063 & 0.007 \end{bmatrix}$$

* Holzmann and Cochran (2012)

Literature Review

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- Matrix Exponentiation
  - MATLAB & SAS → quick…but not readily accessible.
  - Numerically

$$e^{(Q*|A|)} = \sum_{k=0}^{\infty} \frac{(Q*|A|)^k}{k!}$$

  - Diagonalization

$$e^{Q} = S \cdot e^{Q} \cdot S^{-1} \rightrightarrows S \cdot \begin{bmatrix} e^{\lambda_1} & & \\ & \ddots & \\ & & e^{\lambda_n} \end{bmatrix} \cdot S^{-1}$$

* *Strang (2006)*

Methodology

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- Assumptions
- Methods
- Parameters
- Flow Diagram
- Model
- Outputs

Methodology - Assumptions

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- The baseline for an Operations Area (AO) is 1km^3
- # and type of birds known to an acceptable level.
- The velocity of birds in the AO is negligible compared to the speed of the aircraft – bird is stationary wrt aircraft.
- The number of birds in the AO is constant.
- An aircraft/bird encounter within the ellipse formed by the height and width of an aircraft will be considered a strike.
- Altitude distributions of birds (if known) will following a gamma distribution (birds within an altitude band will be distributed via a Poisson process).
- Small Bird Estimations are utilized as needed.

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Methodology - Parameters

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Parameters

Identifier	Description	Used in
L	AO Length in meters	|V| → AO Volume
W	AO Width in meters	|V| → AO Volume
H	AO Height in meters	|V| → AO Volume
A_Mx	Aircraft Altitude Max	|v| → Modified AO Volume
A_Mn	Aircraft Altitude Min	|v| → Modified AO Volume
A_W	Aircraft Wingspan	|A| → Aircraft Volume
A_H	Aircraft Height	|A| → Aircraft Volume
Surface_Area	Aircraft Frontal SA	|A| → Aircraft Volume
A_D	Aircraft Distance	|A| → Aircraft Volume
T	Total # birds in AO	λ → Intensity Function
$B_{species}\alpha$	Gamma Alpha value	$\lambda'_{species}$Alt → Intensity Function modified volume
$B_{species}\beta$	Gamma Beta value	$\lambda'_{species}$Alt → Intensity Function modified volume
$B_{species}T$	% of T	$\lambda'_{species}$ → Intensity Function bird % known
$RH_{species}$	Species Risk Score	$R_{species}$ = total risk by species

Calculations

|V| = L * W * H {length * width * height}
|v| = L * W * (A_Mx - A_Mn) {length * width * altitude band height}
|A| = ½(A_W) * ½(A_H)*)*π*(A_D) {area of ellipse * distance}
λ = T / |V|
$\lambda'_{species}$ = ($B_{species}$T * T) / |V|
$\lambda'_{species}$Alt = [($B_{species}$T * T)*(GammaCDF(A_Mx) – GammaCDF(A_Mn)] / |v|

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Methodology – Flow Diagram

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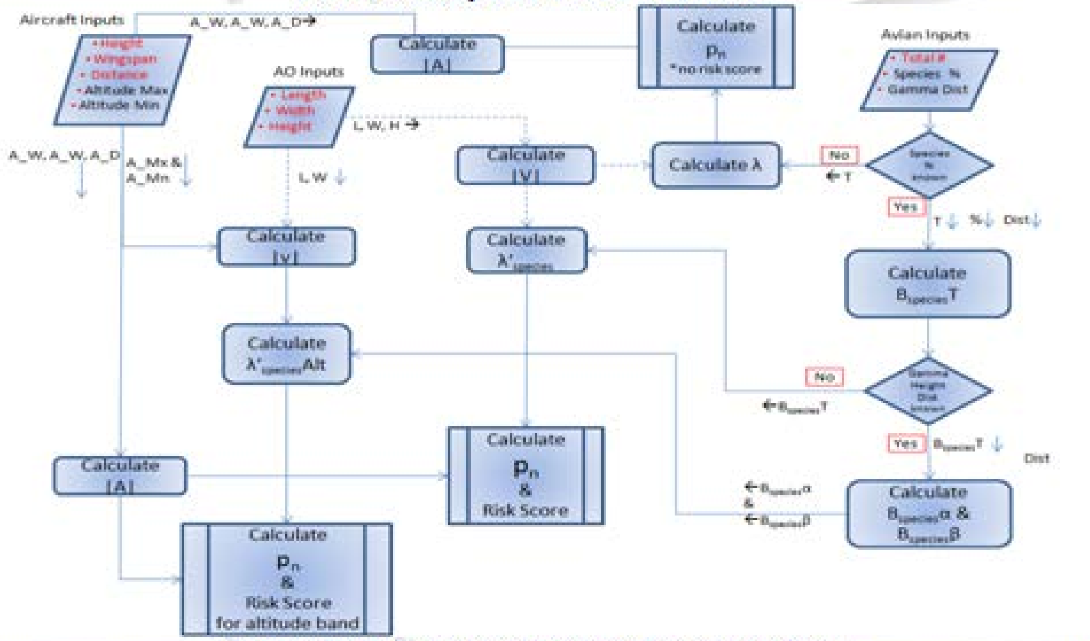

Methodology - Model

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- Model inputs / outputs

	INPUT							OUTPUT	
Case #	AO Volume	Aircraft	Distance Through AO	Aircraft Altitude Band	Total # Birds in AO	Specific Species %	Avian	Strike Probability	Risk Score
1	x	x	x		x			x	
2	x	x	x		x	x		x	x
3	x	x	x	x	x	x	x	x	x

Methodology - Model

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Case #1

1) Calculate hit probability

$$p_n = e^{-\lambda * |A|} * \frac{(\lambda * |A|)^n}{n!}$$

All 3 cases need AO data & Aircraft data inputs. Following are the extra inputs needed by case and the respective output::

Case 1 Input = Total birds
Case 1 Output = Probability of Hit

Case 2 Inputs = Total birds & Species %
Case 2 Output = Hit probability and Risk Score

Case 3 Inputs = Total birds, Species %, Gamma Distribution and Aircraft altitude band
Case 3 Output = Hit probability and Risk Score (fidelity on the altitude band)

Case #2

1) Calculate hit probability per specific species

$$p_{species}(n) = e^{-\lambda'_{species} * |A|} * \frac{(\lambda'_{species} * |A|)^n}{n!}$$

$\lambda'_{species}$ (generated using percentage of specific bird in the AO)

2) Aggregate→ number of birds x probability of hit (eliminating double counts) x species specific risk level

$$R_{species} = \sum_{n=0}^{N} n * p_{species}(n) * RH_{species}$$

3) Sum over all species

$$R = \sum_{\text{All species}} R_{species}$$

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Methodology - Model

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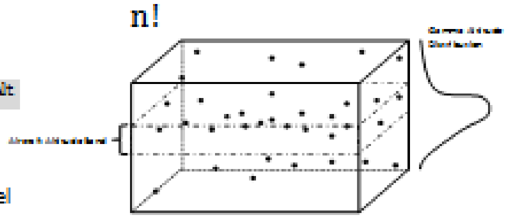
Case #3

1) Calculate hit probability per specific species

$$p_{species}\,Alt(n) = e^{-\lambda'_{species}Alt * |A|} * \frac{(\lambda'_{species}Alt * |A|)^n}{n!}$$

doubly-stochastic process modified intensity parameter $\lambda'_{species}Alt$



2) Aggregate→ number of birds x probability of hit (eliminating double counts) x species specific risk level

$$R_{species}\,Alt = \sum_{n=0}^{N} n * p_{species}\,Alt(n) * RH_{species}$$

3) Sum over all species

$$R = \sum_{\text{All species}} R_{species}\,Alt$$

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Methodology - Inputs

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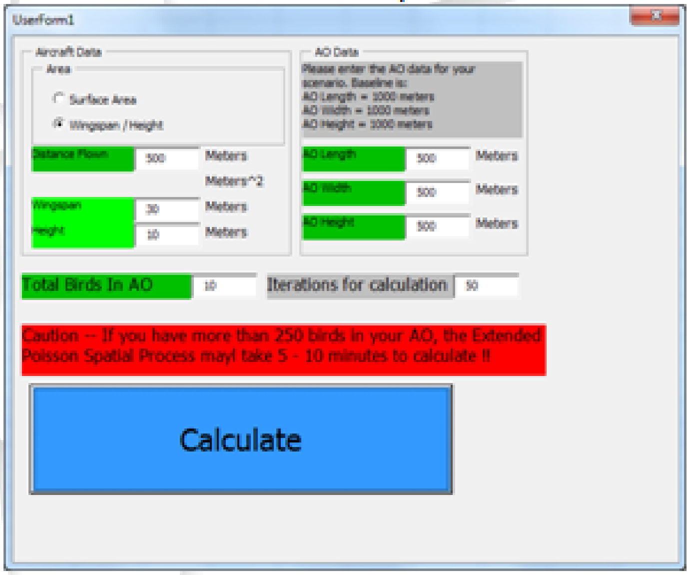
CASE # 1 - Inputs



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Methodology - Outputs

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CASE # 1 - Output

From/To	0	1	2	3	4	5	6	7	8	9	10
0	0.990624	0.009336	0.000040	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
1	0.000000	0.991538	0.008410	0.000052	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
2	0.000000	0.000000	0.992492	0.007485	0.000023	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
3	0.000000	0.000000	0.000000	0.993428	0.006554	0.000019	0.000000	0.000000	0.000000	0.000000	0.000000
4	0.000000	0.000000	0.000000	0.000000	0.994364	0.005623	0.000013	0.000000	0.000000	0.000000	0.000000
5	0.000000	0.000000	0.000000	0.000000	0.000000	0.995501	0.004690	0.000009	0.000000	0.000000	0.000000
6	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.996259	0.003736	0.000005	0.000000	0.000000
7	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.997178	0.002819	0.000003	0.000000
8	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.998118	0.001881	0.000001
9	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.999058	0.000942
10	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	1.000000

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Methodology - Inputs

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CASE # 2 - Inputs

Methodology - Outputs

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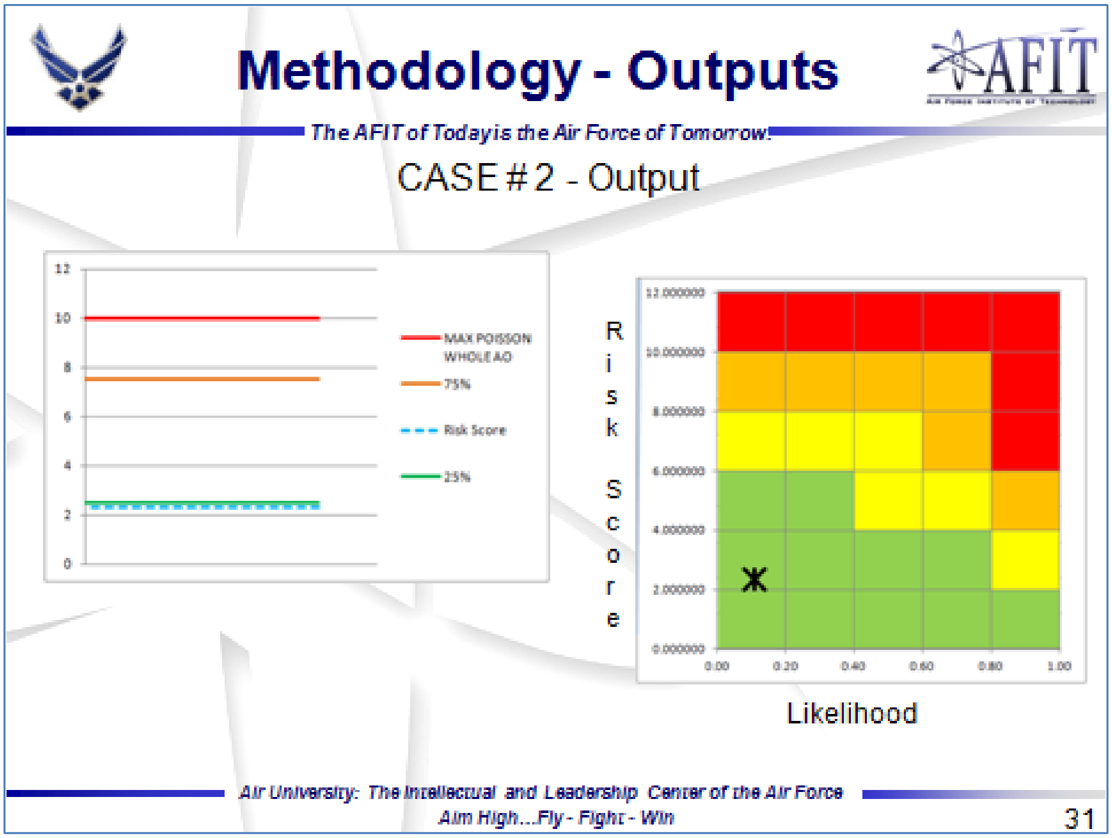
CASE # 2 - Output

Probability of No hit	0.89
Prob of at least 1 hit	0.11
Risk Score (Raw)	1.400

n	P(n)
0	88.9%
1	10.3%
2	0.8%
3	0.0%
4	0.0%
5	0.0%
6	0.0%
7	0.0%
8	0.0%
9	0.0%
10	0.0%
11	0.0%
12	0.0%
13	0.0%
14	0.0%
15	0.0%
16	0.0%
17	0.0%
18	0.0%
19	0.0%


Methodology - Outputs
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CASE # 2 - Output
MAX POISSON WHOLE AO
75%
Risk Score
25%
Risk Score
Likelihood
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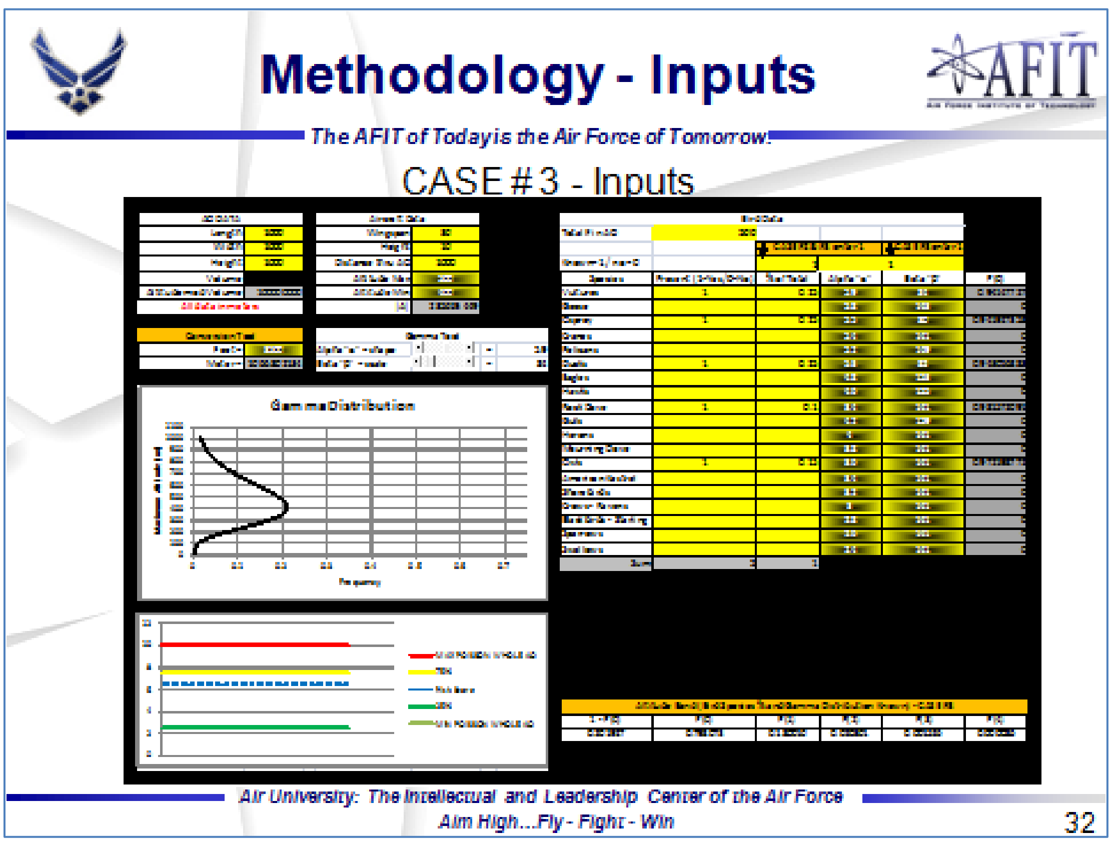
Methodology - Inputs
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CASE # 3 - Inputs
Gamma Distribution
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Methodology - Outputs

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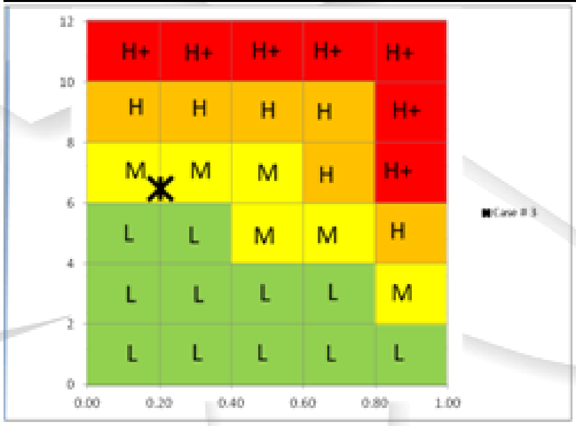
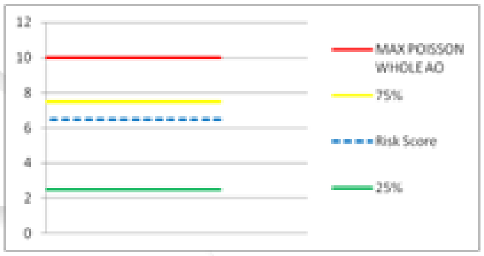
CASE # 3 - Output

Altitude Band (Bird Species % and Gamma Distribution Known) - CASE #3					
1 - P(0)	P(0)	P(1)	P(2)	P(3)	P(4)
20%	80%	18%	2%	0%	0%

P(0) =	80%
Hazard Score =	0.406065





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Analysis

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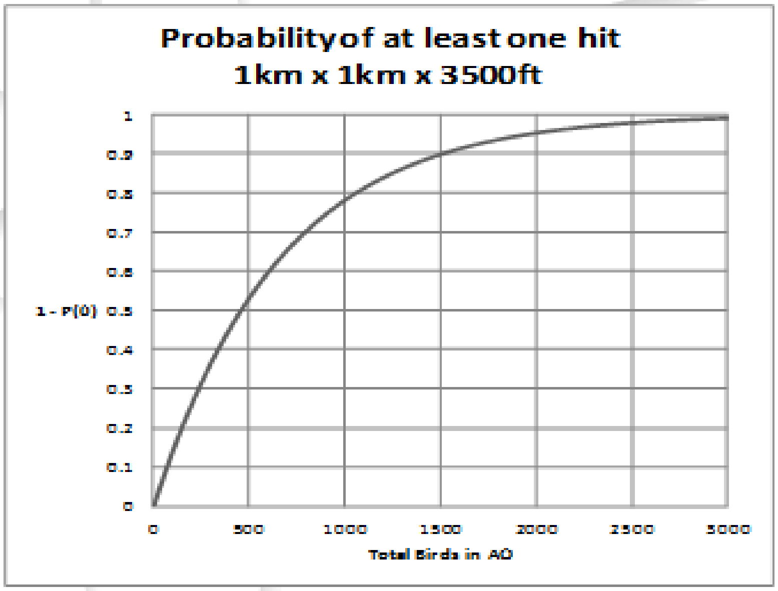

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Analysis

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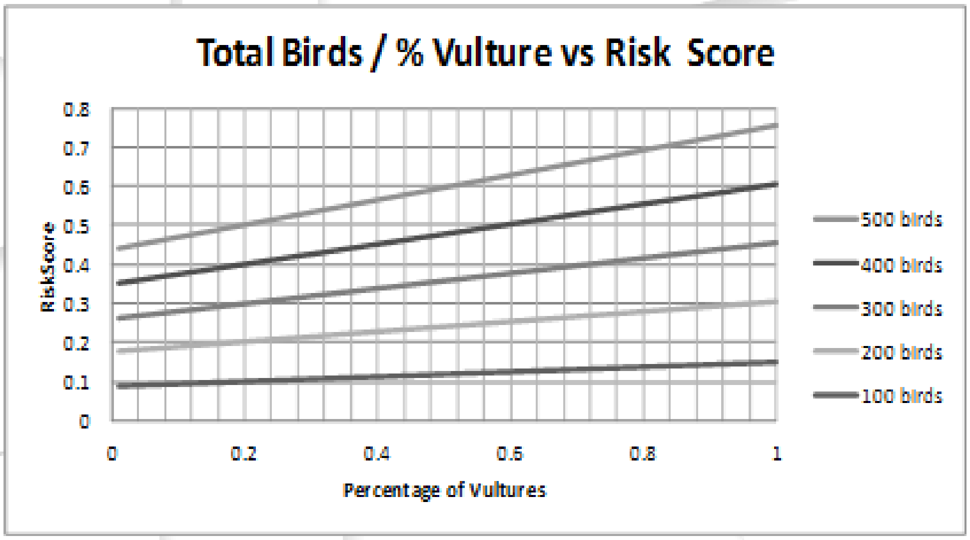

Analysis

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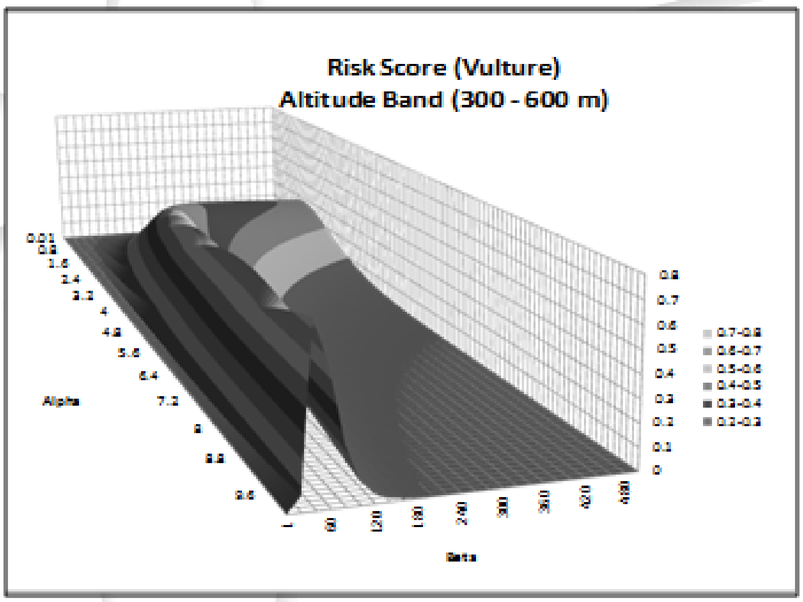

Conclusion

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- The Spatial Poisson Process accounts for the spatial distribution of birds within a bounded area of operations.
- The Extended Spatial Poisson Process allows for over-dispersed and under-dispersed object densities in this bounded space (the AO).
- The Relative Risk Scores account for differing hazards associated with striking different species of birds.
- The Gamma Altitude Distributions provides further fidelity when determining the hazard associated with a particular altitude band to be flown.
- The model allows for varying levels of user input and provides numerous outputs to include collision probabilities, a raw risk score, a standardized and graphical relative risk score, a gamma distribution determination tool and a Risk Filtering and Ranking Method graphic.

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Areas for Future Research

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- Account for the varying flock sizes
  - Shot noise Cox processes and the finite Gibbs point process could be utilized to model the clustering of birds within an AO.

- Flock shapes and density distributions within the flock could be studied. We treat them as random, but there may be structure.

* *Møller & Waagepeterson, (2006)*

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10. Vita

Major Brady J. Vaira graduated as Valedictorian from Lambert High School, Lambert, Montana, in 1992. He earned a Bachelor of Science degree in Mathematics at Rocky Mountain College, Billings, Montana. While at Rocky Mountain College, he was a four year member of the college basketball team, the college flying team and also served as president of the student body his senior year. He graduated in 1996.

Major Vaira continued his education at Montana State University, Bozeman, Montana, from 1996 to 1998 where he earned a Master's of Science Degree in Mathematics. In 1998, he was selected as the graduate mathematics instructor of the year. While at Montana State University, Major Vaira attended ROTC and was commissioned into the United States Air Force in 1998. While in ROTC, Major Vaira was selected as the #1 cadet and a distinguished graduate from his Field Training Course.

Major Vaira attended Air Traffic Control Training at Keesler AFB, Mississippi. He was selected as a distinguished graduate from ATC School. He then attended Airfield Management training at Altus AFB, OK and became a fully qualified Airfield Operations Officer.

Major Vaira's career has spanned the spectrum of the airfield operations career field. He has led air traffic control and airfield management at the USAF's busiest ATC facility (Nellis AFB) and at the most remote airbase (Thule, AB, Greenland). Additionally, he has been stationed at Hickam AFB, Hawaii, were he was a combat airspace manager; Moron AB, Spain, where he was the director of operations for the Air Base Squadron and at Fort Bragg, North Carolina, where he was a combat airspace manager for the Joint Special Operations Command. .

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13. SUPPLEMENTARY NOTES

14. ABSTRACT: Aircraft collisions with avian species are a serious safety problem as well as a serious economic issue. Aircraft / bird strikes have resulted in 33 fatalities, the loss of 39 aircraft, and damages to aircraft in excess of $820M for the United States Air Force. The objective of this paper is to create a closed form mathematical model that estimates the probability of a bird / aircraft collision and provides a risk score that can be utilized to underpin decisions made by planners and pilots. The major components of the model are the spatial Poisson process, the extended spatial Poisson process, a gamma distribution of bird altitudes, a relative risk score, a standardized risk score scale, and a risk filtering and ranking method. The spatial Poisson process allows for an independent distribution of birds within a bounded area. The extended spatial Poisson process accounts for the removal of birds from calculations within the bounded area after they have been encountered. The gamma distribution models the distribution of specific bird altitude bands within a bounded area. The relative risk score is a weighted risk score for 19 different species of birds that an aircraft might encounter. The standardized scale aggregates all risk scores over all the bird species and then calculates the value in a 0 to 10 scale. The risk filtering and ranking model combines the effects of a hit with the likelihood of a hit and displays the result in a graphic. The overall model that combines these components and calculates the output is an original contribution to the field of aircraft / avian collision models. Exercising the model reveals significant factors that influence the risk score associated with flying in a particular area. They are the total number of birds in the bounded region, the mix of species within the bounded region, the size of the aircraft, and the gamma height distribution of the birds within the bounded region. Knowing the gamma height distribution for the specific birds in an operations area (AO) can provide more fidelity to the planner. In fact, in several scenarios where the same number and species of birds for an AO was used, the difference in the overall aggregated risk score was twice as high as the score that was calculated when the gamma height distribution was not known. Additionally, when there were densely populated altitude bands of birds in the operations area, avoiding these bands cut the overall risk score by up to 50%. This is very useful information for decision makers to have when they are planning the specifics of their operations.

15. SUBJECT TERMS: Risk, Collision, Bird / Aircraft, Poisson Spatial Process, Gamma Distribution

16. SECURITY CLASSIFICATION OF:			**17. LIMITATION OF ABSTRACT**	**18. NUMBER OF PAGES**	**19a. NAME OF RESPONSIBLE PERSON** Jeffery K. Cochran, PhD
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